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**OHIO RIVER VALLEY SOILS  
SEMINAR, 1973**

**GEOTECHNICS IN  
TRANSPORTATION ENGINEERING**

**October 5, 1973**

**PROCEEDINGS**

**OHIO RIVER VALLEY  
SOILS SEMINAR, 1973**

**October 5, 1973  
Lexington, Kentucky**

**Presented by**

**KENTUCKY SOIL MECHANICS AND FOUNDATIONS GROUP  
Kentucky Section, American Society of Civil Engineers**



**Cosponsored by**

**Cincinnati - Dayton Soils Group  
American Society of Civil Engineers**

**Department of Civil Engineering  
and  
Office of Engineering Continuing Education  
University of Kentucky**

**Department of Civil Engineering  
University of Louisville**

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## FORWARD

The 1973 Ohio River Valley Soils Seminar was held on October 5, 1973, at the Holiday Inn North in Lexington, Kentucky. This was the fourth annual seminar presented alternatively by the Kentucky Soil Mechanics and Foundations Group and the Cincinnati-Dayton Soils Group. The seminars are presented as a forum for discussion and interchange of ideas and techniques among those practicing the geotechnical disciplines, particularly soil mechanics and engineering, rock mechanics, and geology. The purpose of this fourth seminar was to present a discussion of the principles of geotechnics as applied to the location, design, construction, and maintenance of transportation facilities.

This seminar was principally organized by the Kentucky Soil Mechanics and Foundations Group of the Kentucky Section, American Society of Civil Engineers. Cosponsors of the meeting included the Cincinnati-Dayton Soils Group of the American Society of Civil Engineers, The Department of Civil Engineering at the University of Louisville, and the Department of Civil Engineering and the Office of Engineering Continuing Education at the University of Kentucky. Special thanks for support of the social functions of the meeting are extended to the Mobile Drilling Company, Inc., the Kentucky Foundation Drilling Company, Inc., and the Metal Products Division of Armco Steel Corporation. The planning committee for the seminar consisted of

V. P. Drnevich, Department of Civil Engineering, University of Kentucky;

R. J. Goettle, Richard Goettle, Inc., Cincinnati, Ohio;

D. J. Hagerty, Department of Civil Engineering, University of Louisville;

A. D. May, Fuller, Mossbarger and Scott, Lexington, Kentucky; and

R. C. Deen, Kentucky Bureau of Highways, Lexington, Kentucky, Chairman.

The first presentation by Hagerty, Schmitt, and Pfalzer recognized the value of geotechnical information assembled for one project as it might apply to planning additional future projects. Such geotechnical information assembled by others can be used in site selection, in planning subsurface exploration programs, and in design and construction of earth structures. Sources of such information were identified in the paper and the importance in application of data from such sources was indicated by the authors.

The second paper by Gorman, Hopkins, and Drnevich presented additional information and data concerning the use of the Dutch Cone Penetration Test in residual soils found in Kentucky. It was shown that shear strength parameters developed by the Dutch Cone test are correlated with those obtained by more conventional triaxial methods. Inasmuch as the Dutch Cone test quickly and easily provides large amounts of subsurface information, the technique was suggested as a possible method of much more extensive explorations of specific construction or engineering sites so that the variability of soil characteristics and properties can be more precisely defined for the analysis and design of the engineering structure.

Gedney and Walkinshaw presented in the third paper a discussion of a new concept of earthwork construction. This technique has been used extensively in Europe and is now being implemented on some projects in the United States. The concept of the construction procedure recognizes that soil materials are generally weak in tension. The concept of Reinforced Earth attempts to overcome this deficiency and to provide a reinforced soil "material" that can withstand tensile as well as compressive loadings. The authors indicated the technique has been used most extensively with granular (sandy) type soils and that much research needs to be done in exploring the application of the technique to fine-grained soils.

The analysis of many soil and rock mechanic problems in the transportation engineering field are extremely difficult because of time-consuming manual efforts

required to complete the necessary computations. Huang suggests in the fourth paper that the finite element method has application in geotechnical analyses and is a very powerful tool in providing information as to the performance of a soil or rock structure as a function of time and at various construction stages. However, the author indicated the finite element method requires significant computer facilities. Accordingly, he indicated the importance of carefully selecting situations to be analyzed to avoid obtaining results which are really not worth the expense involved.

Baker introduced a dimension in the analysis of geotechnical structures not often applied at the present time by many practicing engineers. It was indicated in this fifth paper that the optimum design of earth structures must consider possible variations of the earth material as well as variations that occur in the evaluation of the characteristics of these materials. The reliability analysis suggested by Baker would be of great value to the client-owner in deciding between alternate design elements so he has knowledge of the risks.

The sixth paper by McNeal discussed problems and applications of soil engineering to the design and construction of highway facilities as it bears upon decisions of high-level highway administrators. The conflict between desire for more extensive soils exploration and testing was contrasted to the various pressures of time and money brought to bear upon the highway administrator. The paper was particularly interesting inasmuch as the author is not only a chief highway administrator but was educated in and advanced through his profession of the geotechnical disciplines. Accordingly, he is most qualified to discuss points of conflict and agreement between the two positions as it bears upon highway design and construction.

The seventh paper by Wu illustrated by case histories the application of geotechnical information to the analysis of slope stability problems. He discussed potential advantages of the application of geotechnical information in this regard, but he did recognize there are limitations that the practicing engineer and geologist must consider.

Robert C. Deen  
Chairman, Planning Committee  
Ohio River Valley Soils Seminar, 1973

## RETRIEVAL AND USE OF GEOTECHNICAL INFORMATION

D. Joseph Hagerty  
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University of Louisville  
Louisville, Kentucky

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### INTRODUCTION

During the early stages of what is now termed geotechnical engineering, from the time of the Egyptians through the Middle Ages up till the beginning of the 20th century, the mode of operation was empirical. In other words, the approach to design and construction of structures founded on the earth as well as excavations made in the earth was a trial and error procedure. With the development of theories on consolidation, compression, stress distribution, and shear strength of earth materials during the first 50 years of the 20th century, geotechnical engineering obtained a firm theoretical basis. A combination of the experience gained over the past thousands of years, with the theories developed during the early part of the century, has removed much of the uncertainty in designing and constructing structures on and in the earth. However, the material with which the foundation engineer works and the location for the activities of the underground contractor remain essentially mysterious. The practice of geotechnical engineering will always involve a certain amount of uncertainty because of the infinitely varied face of the earth. Any given site is the product of a tremendously long sequence of events; it is a geological product of complicated geologic processes. These processes are continuing at the present date. Therefore, in all cases work in and on the earth must be preceded by efforts to explore and investigate the character of the site for the proposed construction. Thus, an immediate need exists for geotechnical information. A less immediate but equally important need for such information lies in the use of this information together with experience during construction to modify and extend the application of theory to the practice of earth

work engineering.

It is the purpose of this paper to discuss the acquisition and retrieval of geotechnical information. The paper is not intended to be an exposition of methods for library information retrieval; the methods and techniques of data retrieval from technical literature have been described comprehensively and adequately in other publications. Rather, this paper is intended to furnish insights into the need for and benefits from investigations and correlations of actual exploratory data obtained from various sources including governmental agencies and private consulting firms. The major emphasis in this treatment will be upon the need for gathering of data obtained in specific localities by practitioners engaged in developing engineering projects. The benefits to be obtained through the gathering and publication of such data will be described. An example of such an investigation and collection of data will be presented.

### PUBLISHED INFORMATION

A large number of sources for geotechnical information exists within scientific and engineering literature. Access to the technical publications such as the Transactions of the American Society of Civil Engineers, the Canadian Geotechnical Journal, and Geotechnique can be obtained easily through the use of library indices such as the Geodex index. This type of information is familiar to almost every geotechnical engineer and constructor. However, in addition to this type of information, copious data exists in governmental publications such as the Special Reports, topographic quadrangle maps, geologic quadrangle maps and

Memoranda of the United States Geological Survey. Use of such data is fairly widespread in the geotechnical professions; nevertheless, it is quite likely that the scope and breadth of the information available through the United States Geological Survey is not appreciated by most practitioners.

In addition to the information available from USGS, very valuable and extensive information is available through the publications of the United States Department of Agriculture, Soil Conservation Service. The County Agriculture Soil Reports published by the US Department of Agriculture contain very valuable data not only on the occurrence and properties of surficial soils, but on the basic geological setting of the particular county, the distribution of population, the distribution and intensity of agricultural and industrial development in the county, and the climate of that locality. In the following sections further mention will be made of the soil reports.

In addition to the publications of the USGS and the USDA, various technical memoranda and engineering reports of the US Army Corps of Engineers, the US Forest Service, and various other governmental agencies contain information of use in geotechnical engineering. The passage of the National Environmental Policy Act of 1969 has created a requirement for the publication of environmental information concerning every major federal action (activities funded through federal money having a significant effect upon the environment). Included in the assessment of such projects must be a complete description of the project locality prior to the development of the proposed project and a complete description of the impact of the proposed project. Necessarily, a description of the environment of a proposed project will include comprehensive information on the geology of the particular site. Additionally, discussions of the foundation conditions, surficial soil conditions, and other geotechnical aspects of the project will be included in any adequate environmental impact statement. Therefore, these publications constitute a new source of geotechnical information from federal sources.

In addition to the sources of information included within federal governmental activities, many state governmental agencies publish information of considerable utility in geotechnical engineering. The expertly prepared publications of the Division of Research of the Kentucky Bureau of Highways are outstanding examples of the information furnished by state agencies. The publication **Engineering Properties of Kentucky Soils**, for instance, contains a general description of soil conditions throughout the state, with emphasis on transportation engineering applications. In this state, the publications of the Kentucky Geological

Survey also deserve special mention. In neighboring states, excellent sources of information for geotechnical planning and design include the publications of the Illinois Geological Survey. The **Environmental Geology Notes** series of the Illinois Survey contains especially pertinent data useful in many different phases of design and planning. In Ohio, the Ohio Geological Survey has published extensive information on the geotechnical characteristics of a number of areas; the report "Glacial Geology of Wayne County, Ohio" is an outstanding example of a source publication for geotechnical information. In addition, work is now in progress in a number of states on publications concerning the engineering properties of engineering usages of earth materials. For example, the **Field Manual of Soil Engineering** of the Michigan State Highway Department is an excellent handbook detailing the engineering properties of surficial soils in the state of Michigan. Other publications which could be cited include the reports published by the Engineering Experiment Station at the University of Illinois on the engineering properties of soils in Will County, Livingston County, and several other counties in Illinois. Again, the research division of the Kentucky Bureau of Highways has published several very useful monographs and bulletins pertaining to engineering applications of geotechnical information.

On the more localized level, in many cities and towns throughout the United States, municipal agencies accumulate data on subsurface conditions during the progress of their routine investigations. This information is not always published by the agency involved, but it may be obtained from these organizations with very little effort or difficulty. For example, the Metropolitan Sewer District of Jefferson County has furnished to the authors comprehensive data accumulated from a large number of borings made during the course of exploration and installation of sewer lines in Jefferson County. As another example, land use maps and geological maps for Jefferson County, Kentucky, are available from the Louisville and Jefferson County Planning and Zoning Commission. Finally, information on the location of utilities is available from the utility organizations concerned; the Louisville Water Company, the Southern Bell Telephone Company, and similar organizations in this city can furnish valuable data concerning the location of cables, pipes, and other appurtenances. This information is of considerable importance for persons planning excavation below ground level.

In addition to all of the sources mentioned above, special publications or compilations of data by small groups of interested persons must not be neglected. For example, university libraries often contain these

concerning geologic or subsoil conditions in a particular area. As another example, information on the foundation characteristics and the performance of structures may be obtained from the files of the local historical organization. The Filson Club located in the city of Louisville possesses files containing very valuable information on the foundation conditions associated with early buildings within the city. In most cases, the members of such organizations are more than willing to assist the geotechnical engineer or constructor in obtaining data on subsurface conditions at a particular site within the given locality.

## FIELD INVESTIGATION INFORMATION

In addition to obtaining the information from published sources, it is always necessary to obtain data in the field concerning subsurface conditions and the characteristics of earth materials. During preliminary stages of design and construction, the acquisition of overall site data, especially the acquisition of preliminary data concerning virtually virgin territories, may be accomplished best by means of geological reconnaissance, including remote sensing and aerial photography. These techniques have been described in a comprehensive and thorough fashion by Hensey and Barr in a presentation to a previous seminar session in 1971. The focus of this presentation is upon the acquisition of data in a specific locality. In this regard, one of the most fruitful undertakings is a cooperative pooling of information by concerned design professionals. In other words, the mutual accumulation of data on geological and subsurface conditions in a particular area by the geotechnical engineers and constructors operating in that area should be undertaken. This type of activity has been carried out in a number of localities with very beneficial results. For example, an outstanding compilation of data useful to the public has been made in the city of Boston. Beginning with a paper by Worcester in the *Journal of the Boston Society of Civil Engineers* in 1914, the subsoil and foundation conditions of the city of Boston have been very comprehensively described. This operation was carried forward by a committee of the Boston Society of Civil Engineers established in 1920 for the purpose of gathering data concerning the subsoils in Boston. The information was to be gathered and then the committee was ". . . to present it to the Society in such form as to add to the general knowledge and to make it available for reference to any who may wish to get a clear idea of the geological construction of this city." Through the years, beginning in 1931, this committee has published extensive summaries of boring records for

the city of Boston and its surroundings. The latest publication was made available in 1969, through the assistance of the United States Geological Survey. In the combined publications of this Society committee, over 10,000 borings were logged and categorized.

Similar compilations of data on subsurface conditions have been made for the following cities: London, England; Moscow, USSR; Tokyo, Japan; Paris, France; Zurich, Switzerland; and a number of other American and European cities. This type of activity should appear obviously to be of considerable benefit for the public and those engaged in geotechnical engineering and construction. However, certain individuals in the past have been unwilling to engage in this cooperative activity on the grounds that the information obtained through site investigations is the private property of those persons who have paid for the investigation and therefore this information cannot be made available for public use. This type of objection, in the opinion of the authors, constitutes a narrowness of mind and a blindness to the benefits to the public to be obtained through a collective pooling of information. In his recently published book, *Cities and Geology*, Dr. Robert F. Legget, the well-known Canadian engineering geologist, makes this comment on those persons who are unwilling to contribute information to a data pool, "If construction is proceeded with on a site that has been drilled, the excavation itself will reveal far more than the preliminary bore holes and this can readily be seen by anyone interested. What makes this particular objection so weak is that those who do not wish to disclose the results of their borings were in all probability aided in the planning of their own program by the general information available from adjacent sites that have been made available to them."

The benefits to be gained through a cooperative accumulation of geotechnical data about a particular locality will now be illustrated through the example of such an operation for Jefferson County, Kentucky.

## CASE HISTORY JEFFERSON COUNTY, KENTUCKY

In conjunction with graduate research activities at the University of Louisville, the authors undertook a compilation of subsurface data for Jefferson County, Kentucky. In this compilation they were aided by a number of individuals and agency personnel. Particular credit should be accorded to Mr. James Flaig and the staff members of the H. C. Nutting Company, Cincinnati, Ohio; Mr. Milton M. Greenbaum, of Greenbaum and Associates, Louisville, Kentucky; Dr.

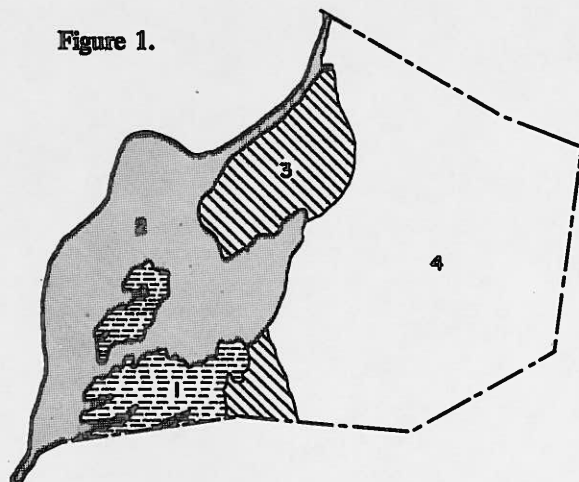


Robert Deen of the Division of Research, Kentucky Bureau of Highways; the staff engineers of the Jefferson County Metropolitan Sewer District; and other individuals who contributed information in this study. Additionally, Prof. Vincent Drnevich contributed freely of his time, attention, and advice in assisting the authors in this task. Through the kind contributions of time, effort, and information by these individuals a comprehensive picture of geotechnical conditions in Jefferson County was obtained. The procedure and results of the investigation for this particular county are presented below.

#### A. Preliminary Literature Search

As the first step in the investigation of Jefferson County subsurface conditions, the research team gathered all available information on the basic geology and physiography of Jefferson County. On the basis of the collected information, the county was divided into four physiographic divisions as shown in Fig. 1. The Outer Blue Grass extends into Jefferson County on the east in the form of limestones, shales and shaley limestones which dip gently to the west. West of the Outer Blue Grass, the erodable shales of Devonian age have been reduced in the Scottsburg Lowland. Near the lowlands, the Ohio River Valley itself widens to a considerable extent; the valley contains glacio-fluvial sands and gravels and recent alluvium near the stream channels. In the areas of valley developed in former flatlands, the deposits are slackwater clays laid down on shale. In the western portion of the county, limestones of Mississippian age are found; they represent

Figure 1.



#### PHYSIOGRAPHIC DIVISIONS

- 1 MISSISSIPPIAN PLATEAUS
- 2 OHIO RIVER VALLEY
- 3 SCOTTSBURG LOWLAND
- 4 OUTER BLUEGRASS

the edge of the Knobstone Escarpment and the Mississippian plateaus.

The bedrock geology of the county is summarized in Fig. 2. The rocks of the Outer Blue Grass consist of Ordovician and Silurian rocks as mentioned. Devonian strata are represented by the widespread New Albany Shale and weak, erodable limestones (principally the Jeffersonville). The Mississippian rocks in the county consist of rather resistant, thick-bedded limestones. Along the river valley the bedrock is overlain with as much as 120 feet of Pleistocene and Recent alluvium. In these areas the unconsolidated granular materials should be considered parent materials for soil development.

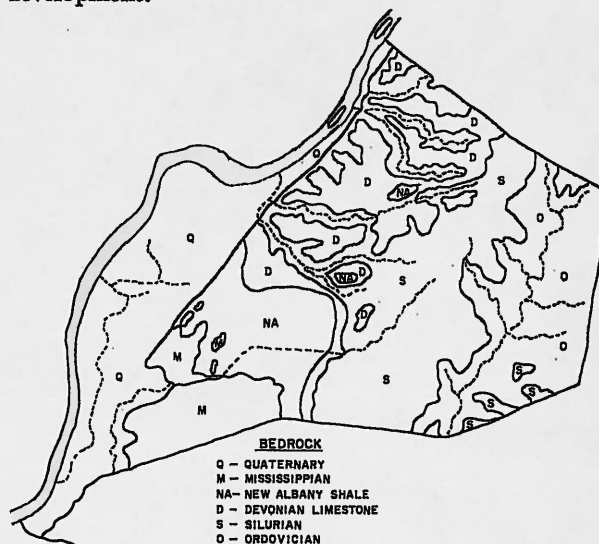


Figure 2.

In the eastern portion of the county along the river valley, a thin covering of loess blankets the residuum produced by weathering of bedrock.

Information on surficial soils was obtained from the Jefferson County Soil Survey published by the Soil Conservation Service. On the basis of parent material, weathering processes and local relief, and the soil characteristics produced by these factors, the area of Jefferson County has been divided into seven soil associations. These soil associations formed the basis for an initial division of the county into seven study areas. It was felt that the engineering properties of soils within given associations would be quite similar (within the association). A portion of the research effort was devoted to testing the validity of this division, on the basis of pedological information.

Additionally, research reports on soil characteristics, distribution and construction problems in the state were obtained from the Kentucky Bureau of Highways Division of Research. A pilot study of

highway construction material resources was also made available to the investigators.

Several cursory treatments of subsurface conditions in Louisville were found in written form but no comprehensive compilation of data or description of conditions was discovered.

#### B. Retrieval of Field Investigation Data

As the next step in the study of Jefferson County, a number of subsurface exploration consultants was contacted. These engineering consultants were asked to grant access to their files of exploration data to members of the research team. Some consultants denied all such access on the grounds that information obtained during a boring/sampling program is the sole possession of the person or agency who commissioned the exploration. Other consultants were more cooperative. Some extremely generous individuals and firms not only furnished access to collected data but made reproduction facilities available free of charge.

In this phase of data collection, more than 1200 boring logs representing over 200 sites were obtained. Almost all of these borings contain soil profile descriptions and Standard Penetration Resistance values. Approximately one-fourth contain grain-size analyses, Atterberg Limit values and other test results. The data were consistent from consultant to consultant and agreed with information previously obtained by the authors (see Figs. 3 and 4).

Next, 91 boring logs were obtained from the Metropolitan Sewer District (of Jefferson County) staff engineers. All of these borings show soil descriptions and SPR values. These borings were made along sewer line routes at intervals of as much as several hundred feet.

Finally, 222 boring logs were obtained from the Kentucky Bureau of Highways. The borings described in these logs were made along the routes of the Jefferson Freeway, Shawnee Parkway (western portion of I 264), and Riverside Expressway (I 64). The Jefferson Freeway logs contain soil descriptions, depth to rock and rock quality (coring) data. About one-fourth contain Atterberg Limit values and grain-size analyses. All 65 of the Shawnee Parkway logs contain all of the categories of information obtained for the Jefferson Freeway borings. No SPR values were obtained in these borings. All 53 logs for the Riverside Expressway contain SPR values, grain-size analyses, and Atterberg Limit values; these logs cover the line of roadway from 13th Street to 35th Street.

All of the highway logs were considered representative of individual sites, with the exception of

the bridge-site boring clusters which were grouped appropriately.

After all boring information had been obtained and collated, field reconnaissance was conducted to verify reported conditions and to investigate the division of the county into pedologically-derived soil areas.

#### C. Analysis and Interpretation of Data

After data had been collected and spot-checked in the field, the boring logs were separated into groups according to the location in the soil association areas shown in Fig. 5. Also, where borings were located along straight lines or near straight lines, cross sections paralleling the lines were prepared consisting of a series of soil profiles. A typical cross section for a detailed group of logs is shown in Fig. 6. A number of such cross sections was prepared.

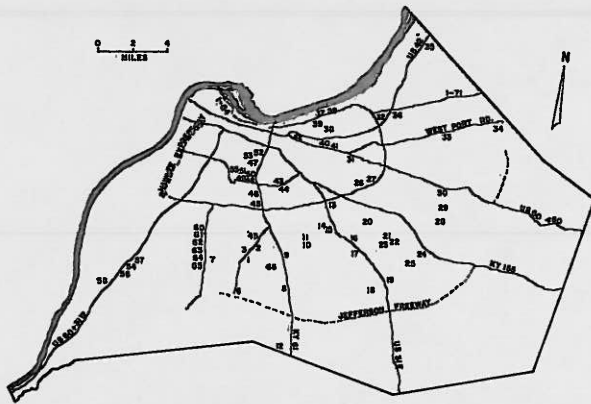
The categorized boring logs were examined in two ways: (1) the soil descriptions in the boring logs were compared with the soil descriptions contained in the Jefferson County Soil Survey, and (2) statistical analyses for mean, standard deviation and variance were made for all the data (SPR values at same depth, grain-size analysis values, and Atterberg Limit values) obtained from the boring logs within each soil area (see Tables 1, 2 and 3).

In general, the soil descriptions coincided quite well for each soil association area. Also, the analysis of variance (together with mean values) indicated that the groupings according to pedological regions was statistically valid. On the basis of these analyses, typical soil/rock profiles were constructed for all areas where a significant number of data were available. The results of the statistical analyses are shown in Figs. 7, 8, 9 and 10. Details of this work are contained in "Engineering Properties of Louisville Subsoils", a thesis submitted for the Master of Engineering Degree by Pfalzer and Schmitt, on deposit in the Engineering Library of the University of Louisville. Additionally, a complete collection of the basic data for this study is on deposit in the Engineering Library.

To render the accumulated data more accessible to interested individuals and to facilitate updating and addition to the initial data bank, the data were transferred to punch cards. A retrieval program was formulated to allow acquisition by geographic location, soil area (I through VIII), or site number.

Finally, to present the compiled data most easily, a subsurface model was constructed. This model was fabricated to allow easy modification or revision, and simple addition of new information.

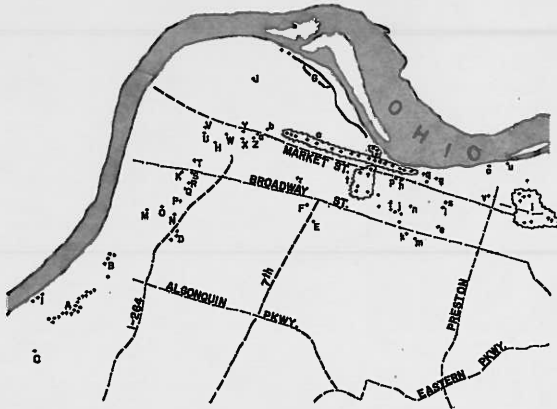
Figure 3.



**SITE LOCATIONS FOR FIG. 3**

1. Grade Lane South of Fern Valley Road
2. Orange Drive and Grade Lane
3. Grade Lane Plant, Reliance Universal
4. Air National Guard, Standiford Field
5. Air National Guard, Standiford Field
6. Outer Loop Adjacent LN Railroad
7. National Turnpike and 3rd Street Road
8. Preston Highway and Markwell Lane
9. Preston and Indian Trail
10. Produce Lane and Poplar Level Road
11. Pinewood Road
12. Mud Lane
13. Bardstown Road and Goldsmith Lane
14. Bashford Manor Mall Road
15. Bacon's Bashford Manor and Bardstown Road
16. Six Mile Lane and Crawford Avenue
17. Raceland Shopping Center, Bardstown Road and Fegenbush Lane
18. Kellwood Subdivision, Ferndale Road
19. Bardstown Road and Fairground Roads
20. Hikes Lane and Klondike Lane
21. Six Mile Lane, Kennedy Estates
22. Six Mile Lane and Huntsinger Lane
23. Six Mile Lane and Huntsinger Lane
24. Eastview and Taylorsville Road
25. Wood Valley Park
26. Dupont Circle Road
27. Dupont Road Bridge
28. Carton Drive and Bunsen Way
29. Bluegrass Industrial Park
30. Shelbyville Road and Mosor Road
31. Shelbyville Road and Fairfax
32. Old Brownsboro Road and US 42
33. Westport and Goose Creek Roads
34. Collins Lane and Westport Road
35. Rose Island Road
36. US 42, Pine Hill Apartments
37. Ohio River and River Road
38. Zorn Avenue, South of I 71
39. Mellwood and Brichwood Road
40. Louisville Water Company Crescent Hill Plant
41. Crescent Hill Road South of Frankfort Avenue
42. Frankfort Avenue
43. Eastern Parkway Extended Care Center
44. St. X Poplar Level Road
45. Kentucky State Fairgrounds
46. Winn Dixie and Locust Lane
47. Municipal Incinerator
48. Floyd and Warnock
49. U of L Interfaith Center
50. U of L Proposed Service Complex
51. U of L Life Science Building
52. Preston and Oak Streets
53. Third and Ormsby
54. Wooddale Drive and Lyneve Drive
55. Algonquin Parkway and Sharp Avenue
56. Johnstontown Road and Dixie Highway
57. Dixie Highway and South Pages Lane
58. Westport Shopping Center
59. Softening Basin, Louisville Water Co. Crescent Hill Road
60. New Cut Road Area
61. New Cut Road Area
62. New Cut Road Area
63. New Cut Road Area
64. New Cut Road Area
65. New Cut Road Area
66. Holiday Towers Apartments

Figure 4.



SITE LOCATIONS FOR FIG. 4

- A. Industrial Force Main, Rubbertown
- B. Fort Southworth Sewage Treatment Plant
- C. Campground Road
- D. Challis Circle
- E. 18th and Breckinridge Streets
- F. 20th and Maple Streets
- G. Riverside Expressway left through 35th Streets
- H. 36th and Michigan Streets
- I. Dupont Facilities
- J. 34th and Gilligan Streets
- K. 40th and Broadway
- L. Ohio River Edge, 2nd to 6th Streets
- M. End of South 43rd Street
- N. End of South Cecil Avenue
- O. End of Garr's Lane
- P. Hale Avenue and Garr's Lane
- Q. 39th Street and Garland Avenue
- R. 806 South 38th
- S. 38th and Broadway
- T. 658 South 38th Street
- U. Riverpark Drive and Amy Avenue
- V. 434 Amy Avenue
- W. 38th and Herman Streets
- X. End of Hermal Street
- Y. 3122 West Jefferson Street
- Z. 225 South 30th Street
- a. 29th and Market Streets
- b. 26th and Main Streets
- c. Main Street, 26th to 6th Street
- d. 12th Street, River to Market
- e. Brook and Chestnut Streets
- f. 7th and Chestnut Streets
- g. 2nd and Main Streets
- h. 7th and Market Streets
- i. Payne and Lexington
- j. 6th and Chestnut Streets
- k. 4th and York Streets
- l. Brook and Liberty
- m. 2nd and York
- n. 5th and Walnut Streets
- o. Ohio River and Big Four Bridge
- p. 8th and Market Streets
- q. 3rd and Main
- r. 22nd and Chestnut
- s. Jefferson and Brook
- t. 9th and 12th Street Watershed
- u. 200 Cable
- v. Preston and Main

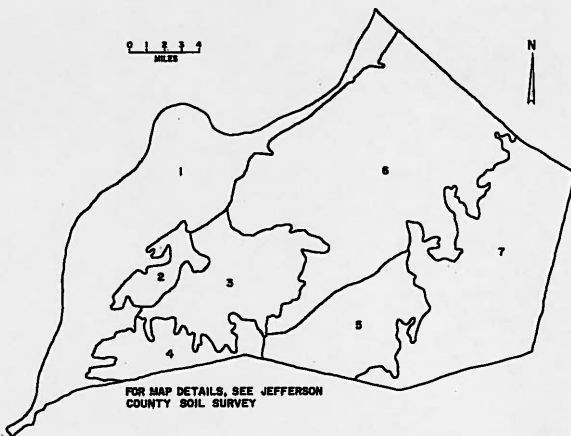


Figure 5.

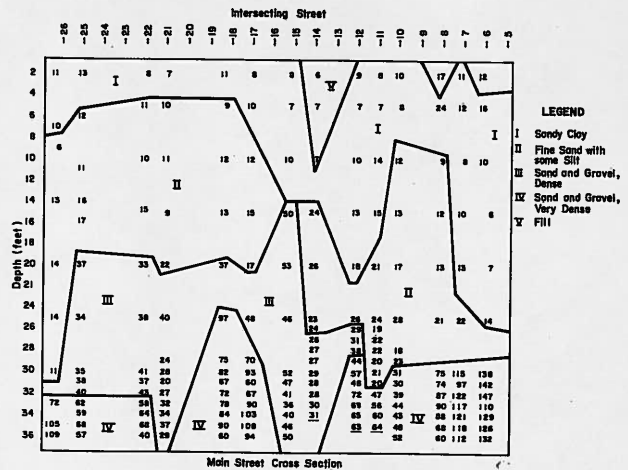
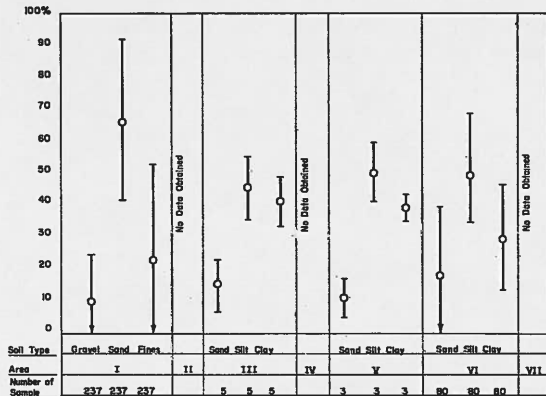


Figure 6.

**TABLE 1**  
**STATISTICAL ANALYSIS DATA OF SPR VALUES**  
**(AT STATED DEPTH)**

GROUP	DEPTH (FEET)	$\bar{X}$	S	V (%)	NUMBER OF SAMPLES
1	5	11.0	7.67	69.2	319
	10	10.7	6.01	56.2	337
	15	11.3	8.31	73.6	327
	20	17.8	15.00	84.3	274
	25	21.2	17.20	81.2	241
	30	28.8	24.30	84.3	208
	35	35.3	28.30	80.2	167
	40	38.6	31.90	82.5	121
	45	31.1	19.20	61.9	81
	50	38.7	21.70	56.0	64
	55	32.8	15.20	46.4	50
	60	31.7	17.20	54.2	40
	65	33.4	16.60	49.8	31
	70	30.9	12.30	39.9	20
	75	22.5	9.37	41.7	15
	80	30.1	14.00	46.5	7
85	43.4	18.80	43.4	5	
90	39.0	17.60	45.1	3	
95	44.0	33.90	77.0	2	
100	59.8	16.90	28.3	4	
2	5	14.5	0.71	4.88	2
	10	12.7	8.75	68.8	3
	15	9.3	6.11	65.8	3
	20	15.3	6.43	35.3	3
3	5	11.9	4.16	34.9	23
	10	12.8	3.87	30.2	14
	15	15.8	3.97	25.1	11
	20	15.3	8.14	53.2	6
	25	10.2	1.75	17.1	4
	30	11.0	4.24	38.5	2
4	No Data Obtained				
5	No Data Obtained				
6	5	10.3	5.01	48.6	67
	10	11.3	7.39	65.4	53
	15	12.4	8.75	70.6	46
	20	12.7	8.89	70.0	32
	25	13.4	16.00	120.0	22
	30	16.3	8.80	54.0	15
7	No Data Obtained				



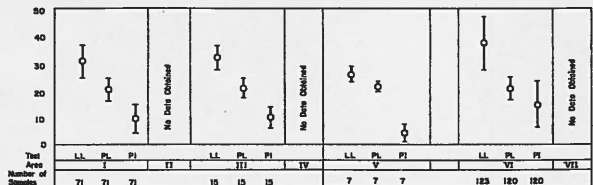
**Figure 7.**

**TABLE 2**  
**STATISTICAL ANALYSIS OF**  
**ATTERBERG LIMIT TEST RESULTS**

GROUP		$\bar{X}$ (%)	S (%)	V (%)	NUMBER OF SAMPLES
1	Liquid Limit	31.1	6.38	20.5	71
	Plastic Limit	20.8	3.11	14.9	71
	Plasticity Index	10.4	5.36	51.7	71
2	No Data Obtained				
3	Liquid Limit	32.7	3.99	12.2	15
	Plastic Limit	21.7	2.95	13.6	15
	Plasticity Index	11.0	4.18	37.9	15
4	No Data Obtained				
5	Liquid Limit	27.0	2.83	10.5	7
	Plastic Limit	21.9	1.22	5.7	7
	Plasticity Index	5.2	2.67	51.9	7
6	Liquid Limit	37.9	9.95	26.2	123
	Plastic Limit	22.0	4.06	18.4	120
	Plasticity Index	15.7	8.03	51.2	120
7	No Data Obtained				

**TABLE 3**  
**STATISTICAL ANALYSIS OF**  
**GRAIN-SIZE ANALYSIS DATA**

GROUP		$\bar{X}$ (%)	S (%)	V (%)	NUMBER OF SAMPLES
1	Gravel	10.4	14.15	136.0	237
	Sand	66.6	25.80	38.8	237
	Fines	22.7	29.60	130.0	237
2	No Data Obtained				
3	Sand	15.2	7.38	48.6	5
	Silt	45.2	9.57	21.2	5
	Clay	41.7	7.16	17.2	5
4	No Data Obtained				
5	Sand	11.3	5.78	51.2	3
	Silt	50.0	9.16	18.3	3
	Clay	38.7	4.16	10.8	3
6	Sand	17.8	22.00	123.0	80
	Silt	51.7	17.00	32.8	80
	Clay	29.7	16.50	55.6	80
7	No Data Obtained				



**Figure 8.**

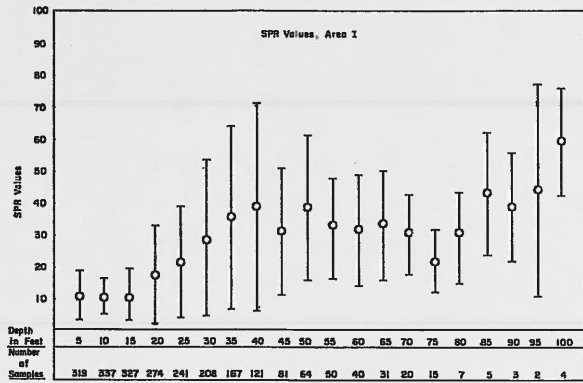


Figure 9.

### CONCLUSIONS

In summary, there exists large amounts of information on subsurface conditions in many areas of the country in published volumes produced by governmental agencies and special groups. Nevertheless, information must always be obtained during the course of any operation in or on the earth. The exploration necessary for such engineering activities may be simplified and made more economical through a compilation of data obtained for the particular area under investigation during the course of exploration and construction for adjacent projects.

The voluntary compilation of such data by concerned professionals constitutes a very worthwhile activity. The data so compiled is valuable during the planning of exploratory programs, during the analysis

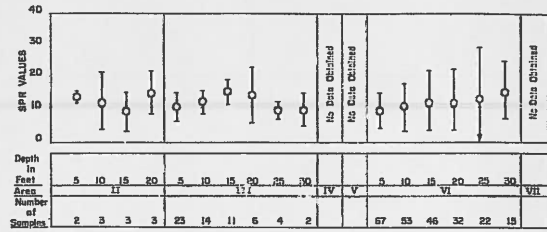


Figure 10.

of obtained exploratory data, and during the assessment of the effects of any particular project upon the surroundings of the affected site.

Information of this sort is extremely useful not only to geotechnical engineers and constructors, but to planners, ecologists, and other persons interested in the maintenance of environmental quality. Compilation of geotechnical data must be an on-going concern in all areas of extensive geotechnical construction. The benefits to be obtained through such activities far outweigh any costs associated with the compilation procedure.

Finally, the information obtained during such activities should be made available in a concise, readable form which is accessible to all interested individuals. Computerized storage and retrieval of such information is an ideal means for such acquisition.

# IN SITU SHEAR STRENGTH PARAMETERS BY DUTCH CONE PENETRATION TESTS

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## INTRODUCTION

The solution of many engineering problems requires an estimation of soil shear strength. In the design of highways, both slope stability analyses and foundation designs are dependent upon correct shear strength input to yield safe, economical solutions. Current methods of estimating shear strength involve removal of a soil sample from its natural state and subsequent laboratory testing in which the in situ conditions are artificially duplicated. This method is both expensive and time consuming.

Elimination of soil disturbance due to sampling and maintenance of in situ stresses on the soil prior to testing can be completely assured only if the sample is tested in place. Dutch cone penetration testing offers the advantage of in situ testing together with a substantial savings in time and money. However, two questions must be answered by engineers in applying Dutch cone data to design problems: 1) Does Dutch cone penetration testing yield a true estimate of in situ shear strength? and 2) How should the results of Dutch cone penetration testing be interpreted? This research was directed towards obtaining some of the answers to these questions.

## CONCLUSIONS

1. In situ shear strength, as measured by triaxial tests, was shown to be approximately 80 percent of the Dutch cone sleeve friction.
2. Further research is needed before Dutch cone data alone can be used as an accurate predictor of soil shear strength. However, the Dutch cone can

predict shear strength when used in conjunction with conventional sampling and testing techniques. It is especially useful in determining the variation of shear strength with depth.

3. Rock fragments can terminate or lead to erratic readings in the Dutch cone penetration test. This tends to limit the sites at which the test can be used, or several penetrations in the vicinity of a given location may be required to obtain data.

## BACKGROUND

The Dutch cone was first manufactured in 1946 at the Soil Mechanics Laboratory of the Technical University at Delf, Netherlands, in conjunction with Goudsche Machinefabriek of Gouda. The cone, shown in Figure 1, is attached to hollow sounding tubes and pushed into the soil to the desired depth. The cone tip is then extended to position B, as shown, by loading the push rods inside the sounding tubes. The load required to advance the cone tip is divided by the tip area ( $10 \text{ cm}^2$ ) and termed cone resistance,  $q_c$ .

In 1953, Begemann (1) suggested the addition of a movable sleeve just behind the cone for the purpose of measuring lateral friction. Adoption of the friction sleeve produced the cone shown in Figure 2. The cone tip is advanced to position B, as before; then the sleeve is engaged and both cone and sleeve are advanced to position C. Data reduction is covered in the section on TESTING PROCEDURE. The Dutch cone penetrometer conforms to the recommendations of the International Society of Soil Mechanics and Foundation (1961) (2) and ASTM proposed test method, "Deep, Quasi-Static Cone Penetration Test" (1973) (2).

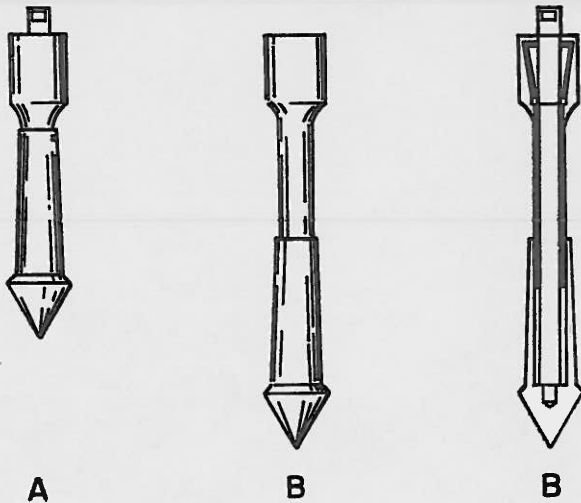


Figure 1. Dutch Cone.

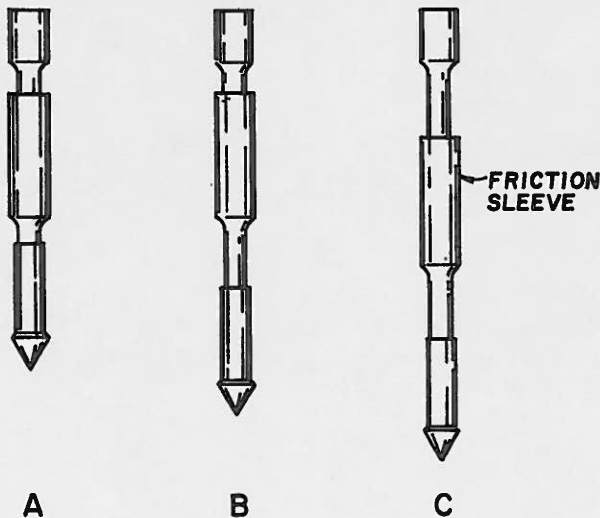


Figure 2. Dutch Cone with Friction Sleeve.

Results of Dutch cone penetration tests have been correlated with standard penetration tests (3, 4, 5), pile load tests (6, 7, 8), plate loading tests (2), and field vane shear tests (9, 10). The Dutch cone has also been used to predict bearing capacity (3), settlement magnitudes (11), and soil type (12, 13, 14).

Determination of in situ shear strength of London clay, using the Dutch cone, was investigated by Thomas (1965) (15). Thomas concluded that "cone penetration resistance of London clay may be interpreted in terms of undrained shear strength" and proposed that cone resistance was approximately 18 times the undrained shear strength.

G. E. Blight (1967) (10) compared shear strengths as determined from unconsolidated, undrained triaxial testing and Dutch cone testing with the shear strengths calculated from stability analyses of landslides. He concluded that the field vane and Dutch cone values compared well, but both overestimated the calculated strengths by a factor of 2. Since the unconsolidated, undrained triaxial testing agreed well with calculated strengths,  $\tau_{uu} = q_c/30$  was recommended for use with the Dutch cone data.

Analytic equations for shear strength parameters,  $c'$  and  $\phi'$ , in terms of bearing capacity factors, shape factors, penetrometer dimensions, and cone resistance were derived by Mitchell (16) in 1973, and model tests were used to verify these equations. Solutions for  $c'$  and  $\phi'$  using these equations involve a trial and error approach.

Correlation of friction resistance, as measured by the Dutch cone friction sleeve, with shear strength was conducted at the University of Kentucky by Cleveland (1971) (14). He concluded that friction resistance was a better measure of in situ shear strength than cone resistance.

#### TESTING PROCEDURE

The Dutch friction cone penetrometer is shown in the various sounding positions in Figure 3. The cone is advanced to the desired depth by loading only the hollow sounding tubes. The penetration of cone tip and sleeve is accomplished by loading the push rods. The cone tip is first extended 4 cm, then the cone tip and sleeve are extended an additional 4 cm. The loads required for tip, and tip plus sleeve penetration, are indicated by the load cell shown in Figure 4. The load cell can be easily mounted on a drill rig, as shown by Drnevich (5).

By loading the hollow sounding tubes, as before, the cone is both retracted to its original position and advanced to a new depth, where the testing procedure





Figure 3. Dutch Cone with Friction Sleeve in the Various Sounding Positions.

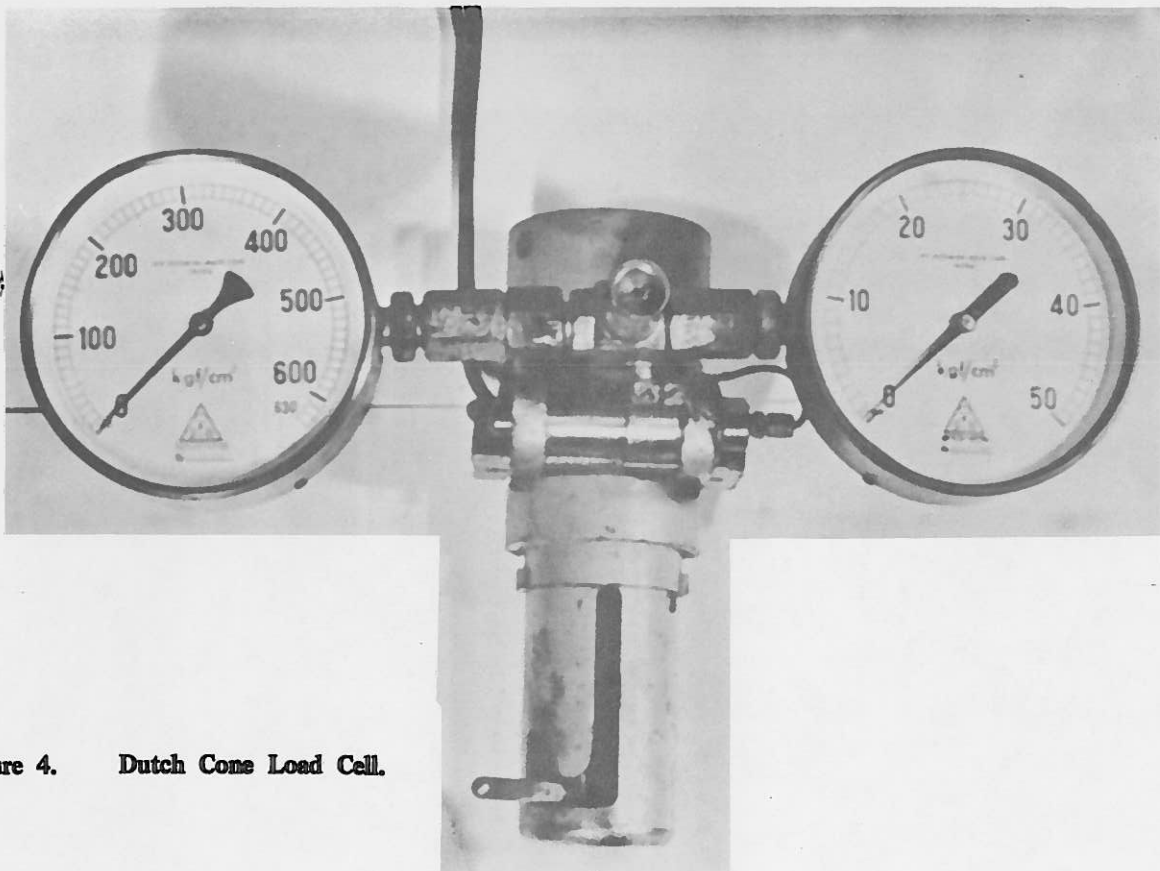
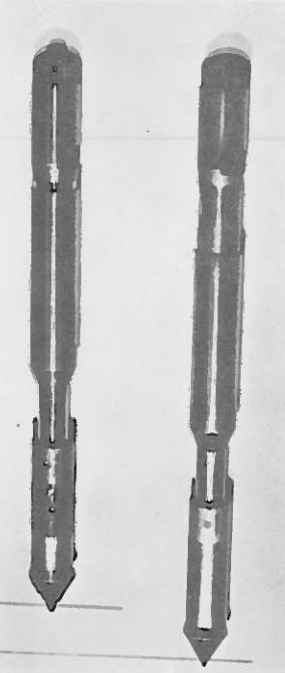


Figure 4. Dutch Cone Load Cell.

is repeated. The vertical distance between individual soundings is usually 20 cm; however, continuous sounding data may be obtained, if desired.

By subtracting cone tip load from cone tip plus sleeve load, the sleeve load is obtained. The load required for cone tip advancement is divided by the cone area ( $10 \text{ cm}^2$ ) and the load required for sleeve advancement is divided by the sleeve area ( $150 \text{ cm}^2$ ) to yield cone resistance,  $q_c$ , and friction resistance,  $f_s$ , respectively. A third quantity, friction ratio (FR), is the ratio of friction resistance to cone resistance. These quantities may then be graphically displayed by plotting them versus depth, as shown in Figure 5.

The base of the cone tip and the friction sleeve have the same diameter. Thus, the resistance

encountered by the friction sleeve,  $f_s$ , is due to friction acting on a cylindrical soil-steel interface. The rate of penetration is constant (2 cm/s) (2). This rate suggests that undrained conditions exist during testing.

No method of test has yet been developed to yield a positive determination of in situ shear strength. The most widely accepted method of determining shear strength parameters is the laboratory triaxial test. However, in situ stresses on the sample are removed in the sampling process and some sample disturbance is inevitable. Although the results of triaxial testing are not absolute, it is widely used and accepted. Therefore, Dutch cone penetration test results were correlated with shear strength by triaxial testing.

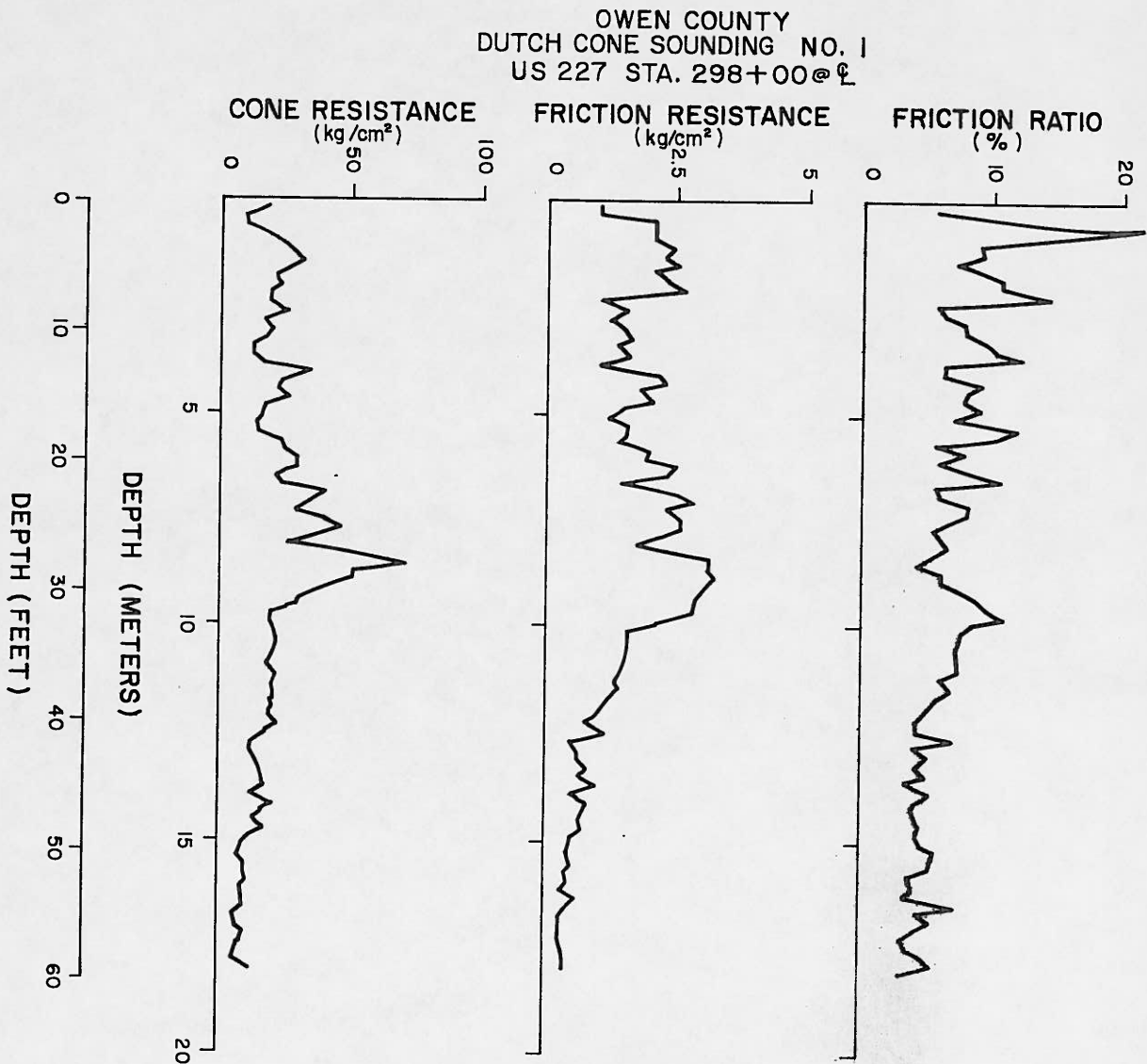


Figure 5. Profile of Dutch Cone Data.

The common strength parameters,  $c'$  and  $\phi'$ , as determined by triaxial testing, are not sufficient to define the shear stress on the failure plane. These parameters define a failure envelope in terms of effective stress. The actual shear stress at failure is dependent upon the normal effective stress on the failure plane and is unknown. To overcome this problem, initial in situ conditions were duplicated for one sample of each set of triaxial tests by consolidating it to the mean in situ effective stress. For soils investigated, an estimate of  $K_0$  was made using Figure 20.8 from Lambe and Whitman (17). An overconsolidation ratio of approximately 1.5 was assumed. This gave a  $K_0$  of approximately 0.62. Consequently, the isotropic confining pressures in the triaxial test were calculated from

$$\sigma'_{\text{consol.}} = (1 + 2 K_0) \sigma'_v / 3, \quad (1)$$

yielding

$$\sigma'_{\text{consol.}} = 3 \sigma'_v / 4. \quad (2)$$

Following isotropic consolidation, the drainage lines were closed and the sample loaded axially, thereby reproducing undrained failure.

An example of a plot of triaxial test data is shown in Figure 6. Note that the stress path method (18, 19) is used to show the continuous stress change during loading. A stress path is the locus of points with coordinates  $p' = (\sigma'_1 + \sigma'_3)/2$  and  $q = (\sigma'_1 - \sigma'_3)/2$ , as calculated from triaxial test data. A Mohr circle, of radius  $q$ , with center at  $(p', 0)$  can be drawn for each point in the stress path. At least one point on a triaxial stress path corresponds to a condition of failure. Triaxial tests at other confining pressures produce independent stress paths, each also having at least one point corresponding to a condition of failure. A line through the points of failure is the  $K_f$  line.

Tests under in situ conditions were used to determine the shear stress on the failure plane,  $\tau_f$ . By assuming in situ failure stresses are mobilized when the in situ stress path intersects the  $K_f$  line, the Mohr circle at failure can be defined from the point of intersection. This point has coordinates  $p'_f = (\sigma'_{1f} + \sigma'_{3f})/2$  and  $q_f = (\sigma'_{1f} - \sigma'_{3f})/2$ . The value of  $\tau_f$  may be determined from  $q_f$  and  $\phi'$ . Derivation of the equation

$$\tau_f = q_f \cos \phi' \quad (3)$$

is shown in Figure 7. Since lines BC and CD are radii of the Mohr circle, they are both equal to  $q_f$  in length.

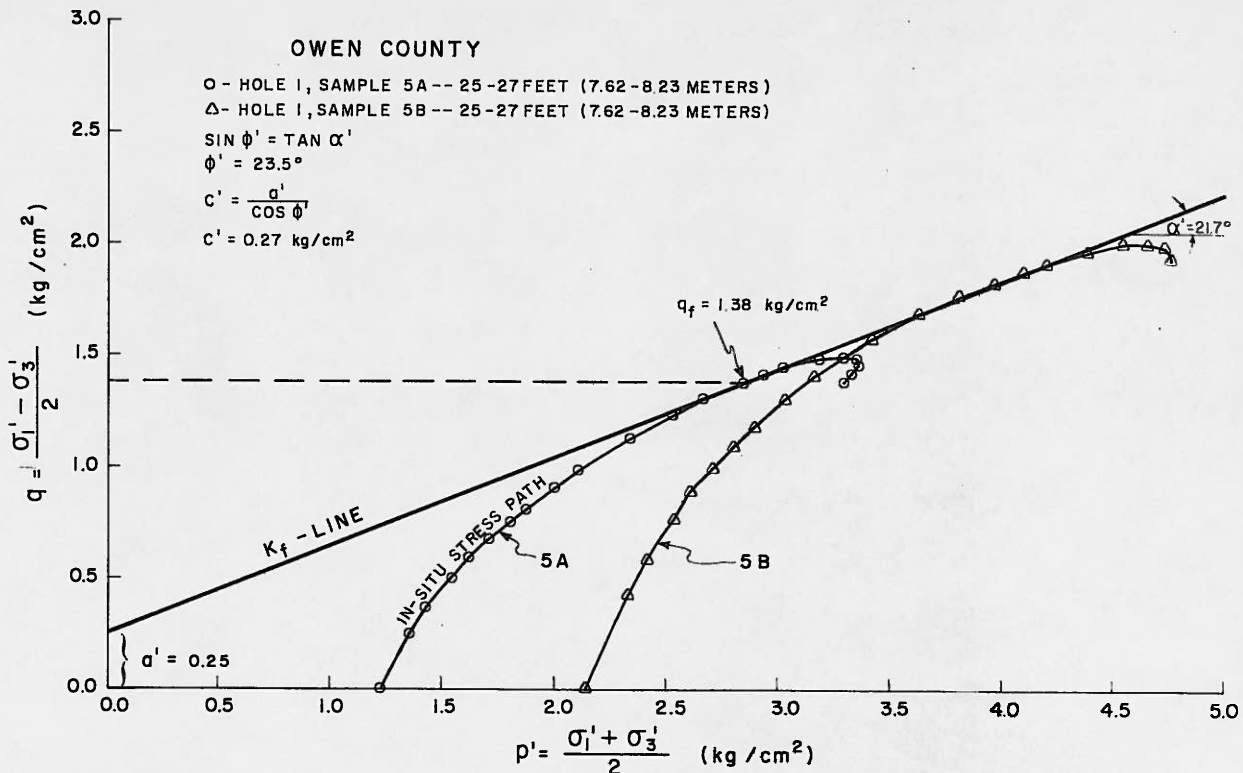


Figure 6. Sample Plot of Triaxial Data.

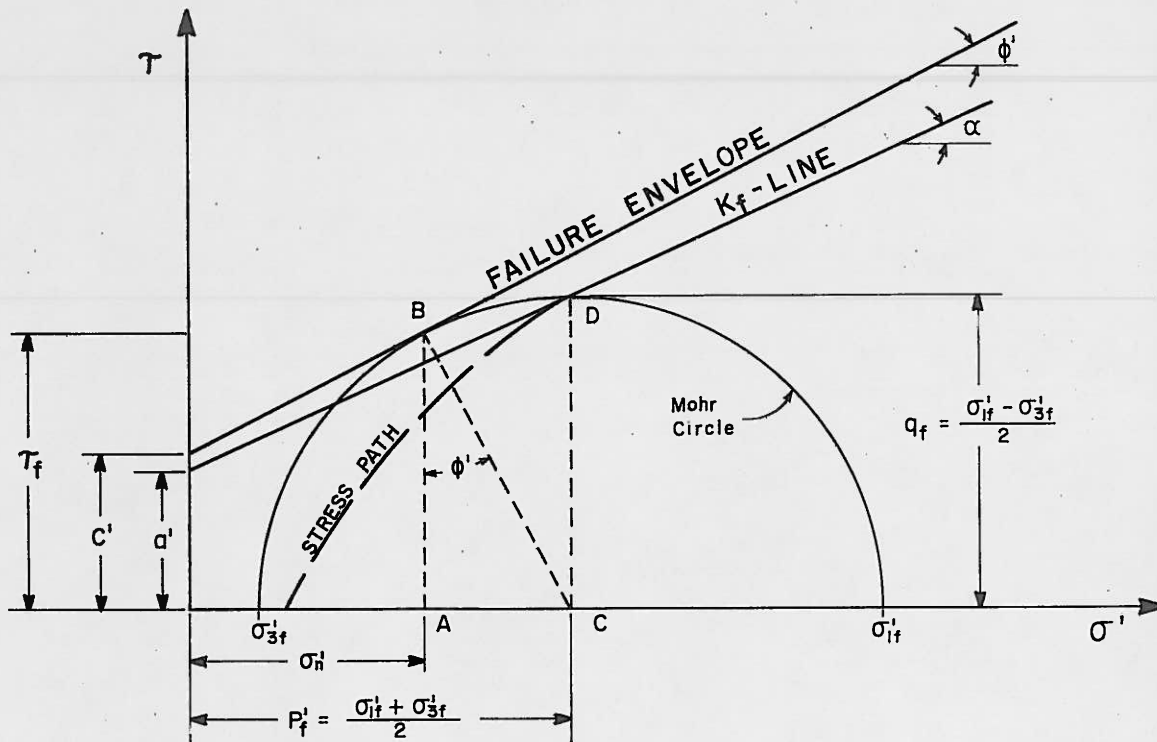


Figure 7. Derivation of the Equation  $\tau_f = q_f \cos \phi'$ .

Line AB is equal to  $\tau_f$  in length. The angle formed by the intersection of AB and BC is  $\phi'$ . The above information applied to the right triangle, ABC, yields the solution for  $\tau_f$ .

Combining data from the research reported herein with Cleveland's data resulted in a regression equation of  $f_s = 1.24 \tau_f$  (see Figure 9).

## RESULTS

The four sites investigated in this research are described in the APPENDIX, and index properties of the soils encountered at these sites are shown in Table 1. Results of Dutch cone penetration testing and triaxial testing performed on undisturbed samples from these sites are summarized in Table 2. A statistical analysis of the data produced a regression line with the equation  $f_s = 1.28 \tau_f$  to describe the data as shown in Figure 8.

Work done by Cleveland (14) in 1971 yielded similar results. When subjected to the same statistical analysis, Cleveland's data resulted in an equation of  $f_s = 1.19 \tau_f$ . However, Cleveland reproduced in situ conditions in an unconsolidated, undrained triaxial test by applying stresses equal to the full overburden pressure to the sample. In this research, in situ conditions were reproduced in a consolidated, undrained triaxial test by applying effective stresses equal to 3/4 of the overburden pressure.

## DISCUSSION

Experimental scatter may be expected in both triaxial and Dutch cone testing. Triaxial test scatter can be caused by disturbances during sampling and trimming of the specimen and vertical variation in the soils tested for a given set of triaxial data. In situ conditions were "duplicated" in the triaxial test by isotropic consolidation of the specimen using a value of  $K_0$  equal to 0.62. Lateral in situ stresses are difficult, at best, to predict and most certainly varied for the soils tested.

Dutch cone soundings were taken at various distances from the bore holes from which the undisturbed samples were taken. Any lateral variation in soil properties could lead to a variation in shear strengths, which in turn could produce scatter unrelated to the test methods.

As can be seen from the example plot in Figure 5, values of  $f_s$  are not constant over the depth interval (0.62 - 0.94 m or 2 - 3 ft) of most samples. However, scatter in values of  $f_s$  as compared with values of  $q_c$  is considerably less. (This is one distinct advantage to

TABLE 1

**INDEX PROPERTIES OF SOILS AT THE  
FOUR TEST SITES**

LOCATION	DEPTH (FEET)	DEPTH (METERS)	LIQUID LIMIT	PLASTICITY INDEX	NATURAL MOISTURE CONTENT (%)	CLASSIFICATION		GRADATION (%)
						UNIFIED	AASHO	
Owen Co.	0 - 40	0.0 - 12.2	40	19	20	CL	A-7-6(20)	Sand - 4 Silt - 46 Clay - 50
Owen Co.	40 - 60	12.2 - 18.3	37	16	27	CL	A-6(16)	Sand - 3 Silt - 56 Clay - 41
Fayette Co.	0 - 30	0.0 - 9.1	36	18	24	CL	A-6	Sand - 20 Silt - 39 Clay - 41
Boyd Co.	0 - 45	0.0 - 13.7	35	14	11	CL	A-6	Sand - 19 Silt - 41 Clay - 40
Lawrence Co. Borehole 11A	15 - 25	4.6 - 7.6	21	1	20	SM	A-4	Sand - 60 Silt - 26 Clay - 14
Lawrence Co. Borehole 8A	0 - 20	0.0 - 6.1	24	4	21	ML - CL	A-4	Sand - 45 Silt - 36 Clay - 19

the use of the friction sleeve over the cone readings for shear strength interpretation.) The value of  $f_s$  used in correlation with  $\tau_f$  is an average of  $f_s$  values obtained at the same depth interval from which the sample was taken.

Correlations shown in this report indicate the Dutch cone can be used as an indicator of in situ shear strength. Dutch cone testing can be performed quicker and easier than boring and sampling and can provide a continuous profile of in situ conditions rather than data obtained from samples at selected intervals.

One major disadvantage to the use of the Dutch cone penetrometer in Kentucky soils is the fact that rock fragments (36 mm or larger), when encountered by the cone tip, will produce erratic readings (2) or halt penetration entirely. In many cases, several soundings in the vicinity of a given location have to be made to obtain the entire profile. Since the major portion of Dutch cone penetration testing in this research was performed on highway embankments, rock fragments were frequently encountered and full penetration was often impossible.

### RECOMMENDATIONS

1. This research has shown the shear strength of soils investigated, as measured by triaxial tests, is approximately 80 percent of the sleeve friction. For increased confidence, the correlation for a given site should be established by performing triaxial tests on samples from that site.
2. The Dutch cone should be used in soils that are relatively free from rock fragments.
3. Further research should be directed toward: 1) assessing the effect on  $\tau_f$  of variations in triaxial test procedure, 2) better duplication of in situ stresses in the triaxial test through a better estimate of  $K_o$ , and 3) accumulation of correlation data on specific soil types.

TABLE 2

## SUMMARY OF TRIAXIAL AND DUTCH CONE DATA

SITE	TRIAxIAL DATA					DUTCH CONE DATA			
	BOREHOLE NO.	SAMPLE(S) TESTED	DEPTH (FEET)	DEPT** (METERS)	$\tau_f$ (kg/cm <sup>2</sup> )	SOUNDING NO.	LOCATION	$f_s$ (kg/cm <sup>2</sup> )	$q_c$ (kg/cm <sup>2</sup> )
Owen Co.	1	3A, 3B 4A	15 - 17 20 - 22	4.6 - 5.2 6.1 - 6.7	1.08	1	5'(1.5 m) from BH No. 1	1.73	22.7
Owen Co.	1	5A, 5B	25 - 27	7.6 - 8.2	1.27	1	5'(1.5 m) from BH No. 1	2.23	42.0
Owen Co.	1	6A, 6B	30 - 32	9.1 - 9.8	0.97	1	5'(1.5 m) from BH No. 1	2.58	26.7
Owen Co.	1	7A, 7B, 7C	35 - 37	10.7 - 11.3	1.18	1	5'(1.5 m) from BH No. 1	1.38	20.3
Owen Co.	1	9A 11A, 11B, 11C	45 - 47 55 - 57	13.7 - 14.3 16.8 - 17.4	0.51	1	5'(1.5 m) from BH No. 1	0.38	12.5
Owen Co.	1	10A 11A, 11B, 11C	50 - 52 55 - 57	15.2 - 15.8 16.8 - 17.4	0.95	1	5'(1.5 m) from BH No. 1	0.40	9.7
Fayette Co.	1	2A, 2B	15 - 17	4.6 - 5.2	1.03	3	3'(0.9 m) W of BH No. 1	1.31	25.7
Fayette Co.	1	3A, 3B, 3C	20 - 22	6.1 - 6.7	0.84	3	3'(0.9 m) W of BH No. 1	1.11	26.7
Fayette Co.	1	4A, 4B	25 - 27	7.6 - 8.2	1.11	3	3'(0.9 m) W of BH No. 1	1.69	49.3
Fayette Co.	2	1A 2A	8 - 10 15 - 17	2.4 - 3.0 4.6 - 5.2	0.89	4, 5, 6	7.5'(2.3 m) W, 4'(1.2 m)W & 3'(0.9 m)E of BH No. 2	1.20	26.9
Boyd Co.	1	2C 3A, 3B	10 - 12 15 - 17	3.0 - 3.7 4.6 - 5.2	1.45	3	67.5'(20.6 m) E of BH No. 1	1.18	29.5
Boyd Co.	1	4B, 4C 5A	20 - 22 25 - 27	6.1 - 6.7 7.6 - 8.2	1.46	3	67.5'(20.6 m) E of BH No. 1	1.62	31.9
Boyd Co.	2	5C	25 - 27	7.6 - 8.2	0.99	1	15.5'(4.7 m) W of BH No. 2	1.57	27.5
Boyd Co.	2	7B, 7C	35 - 37	10.7 - 11.3	1.60	1	15.5'(4.7 m) W of BH No. 2	2.00	41.0
Lawrence Co.	8A	1B, 1D	5 - 7	1.5 - 2.1	0.68	8	5'(1.5 m) S of BH No. 8A	0.91	19.3
Lawrence Co.	8 8A	12 2C	12 - 14 10 - 12	3.7 - 4.3 3.0 - 3.7	1.12	8	5'(1.5 m) S of BH No. 8A, 12'(3.7 m) SE of BH No. 8	0.91	39.0
Lawrence Co.	11 11A	17, 21 1A	17 - 18 21 - 23 15 - 17	5.2 - 5.5 6.4 - 7.0 4.6 - 5.2	0.72	11	65'(24.9 m) S of BH No. 11 & 5'(1.5 m) S of BH No. 11A	0.90	33.2

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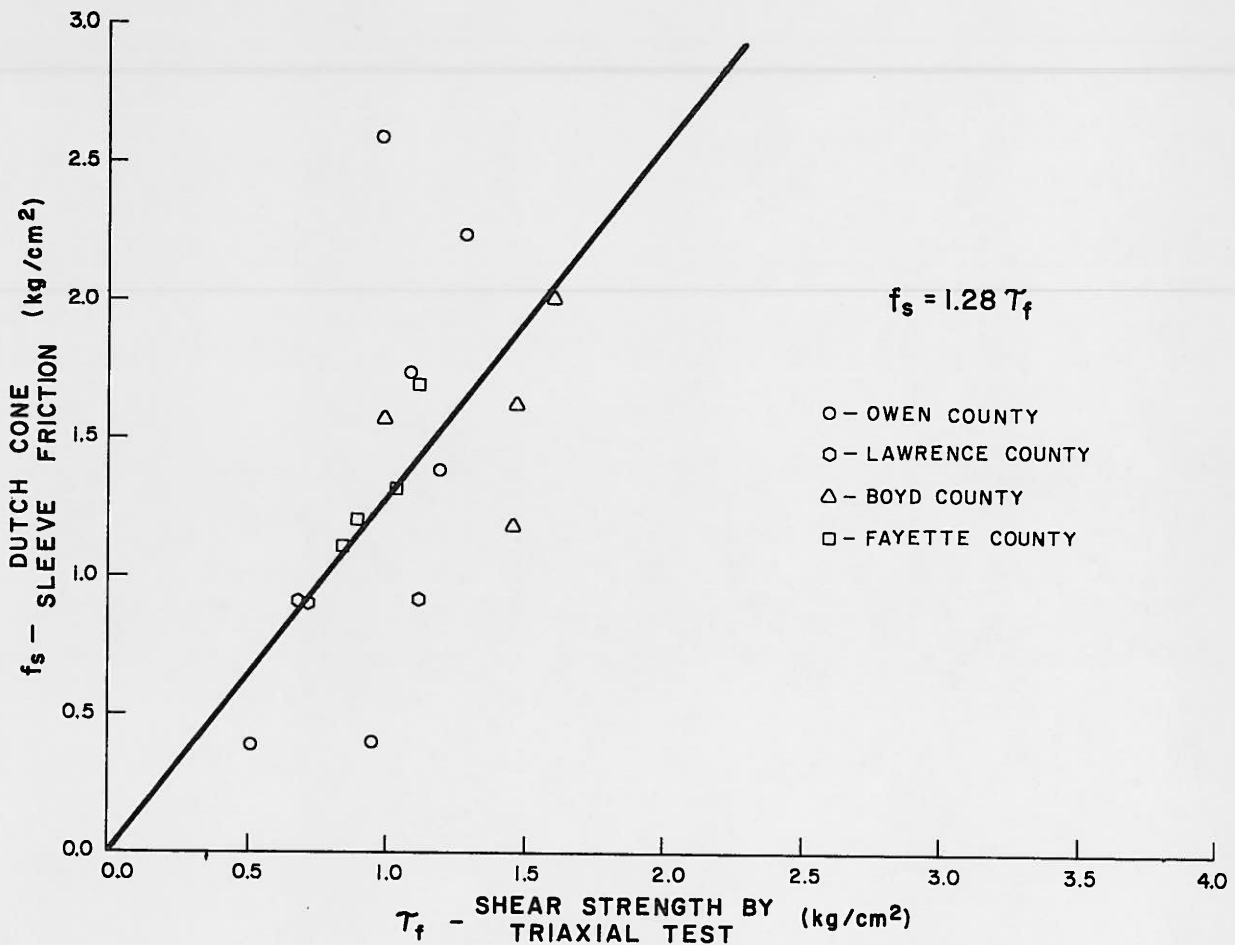


Figure 8. Dutch Cone Sleeve Friction Versus Shear Strength by Consolidated, Undrained, Triaxial Tests.

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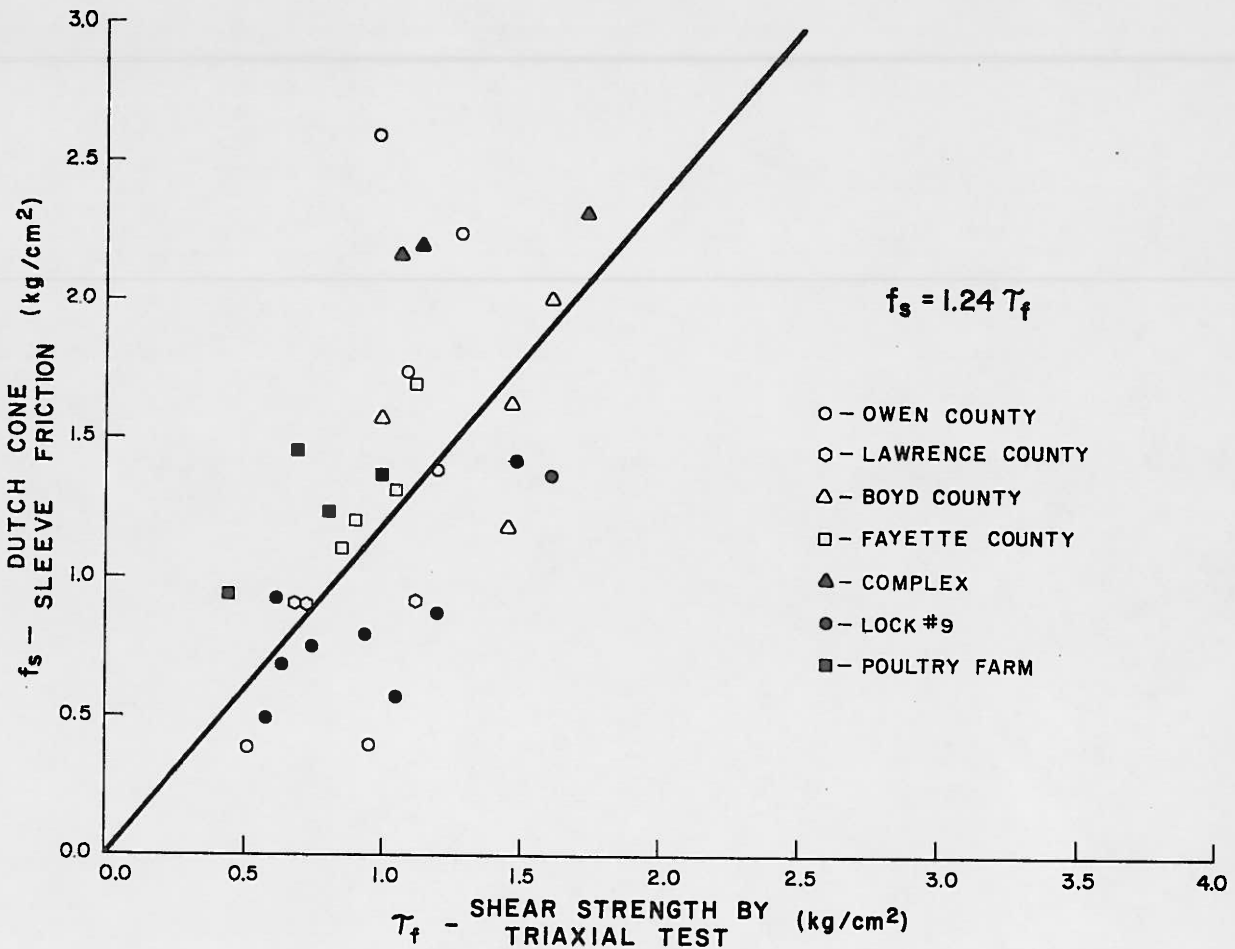


Figure 9. Dutch Cone Sleeve Friction Versus Shear Strength by Triaxial Tests (Cleveland's (14) Data Included).

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## NOTATION

$c'$	= cohesion intercept based on effective stresses
$f_s$	= friction resistance
$K_o$	= coefficient of lateral stress at rest = $\sigma'_1/\sigma'_3$
$p'_{1f}$	= $(\sigma'_{1f} + \sigma'_{3f})/2$
$q_f$	= $(\sigma'_{1f} - \sigma'_{3f})/2$
$q_c$	= cone resistance
$\sigma'_{\text{consol.}}$	= effective consolidation pressure based on the mean principal effective in situ stress
$\sigma'_{1f}$	= major principal effective stress at failure
$\sigma'_{3f}$	= minor principal effective stress at failure
$\sigma'_n$	= normal effective stress on the failure plane at failure
$\sigma'_v$	= effective overburden stress
$\tau_f$	= shear stress on the failure plane at failure
$\tau_{uu}$	= shear strength from unconsolidated, undrained triaxial tests
$\phi'$	= friction angle based on effective stresses

## APPENDIX

### SITE DESCRIPTIONS

#### Owen County

Location: US 227, approach embankment to Eagle Creek Bridge

Date of construction: August 1972

Date of thin-wall tube sampling: September 1972

Date of Dutch cone sounding: October 1972

#### Fayette County

Location: KY 4, approach embankment to Parkers Mill Road Bridge

Date of construction: 1965

Date of thin-wall tube sampling: October 1972

Date of Dutch cone sounding: November 1972

#### Boyd County

Location: I 64, Milepost 188, eastbound lane embankment

Date of construction: 1965

Date of thin-wall tube sampling: March 1973

Date of Dutch cone sounding: March 1973

#### Lawrence County

Location: US 23, 10 to 12 miles (18 to 22 kilometers) south of Louisa, Kentucky (Sta 387+00 - 390+00)

Date of thin-wall tube sampling: February 1973

Date of Dutch cone sounding: May 1973

## REINFORCED EARTH - DESIGN AND CONSTRUCTION

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### INTRODUCTION

"It's costing too much to build highways." "Our construction dollar no longer buys what it has in the past." I'm sure that these are familiar statements to all of you. However, as engineers and constructors, we must heed these words, and we must anticipate even stronger emphasis on costs in the future. Finding new and innovative ways to cut costs commensurate with a quality end product is a primary part of our job. We think we have something that offers just this answer in a construction material called Reinforced Earth.

### GENERAL

The Federal Highway Administration has embarked on a serious campaign to promote the use of research results which can improve the highway design and construction processes. As part of this program, we are constantly looking for new ideas and new methods. Some time ago, we learned about Reinforced Earth. After thorough investigation, we are convinced that Reinforced Earth is an engineering material which highway departments can use in high quality construction with substantially reduced costs over competitive alternates.

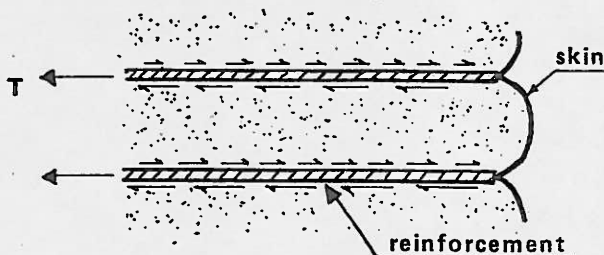


Figure 1.

### DESIGN

What is Reinforced Earth? As the name implies, it is a designed structure whereby a soil is internally reinforced with metal strips so as to form a friction transfer mechanism between soil and reinforcement. This frictional transfer allows the soil-strip mass to exhibit tensile strength under exterior imposed loadings (Figure 1).

Since the design principle is based on friction between soil and strip as well as between soil and soil between strips, the earth used within the reinforced prism must have certain basic engineering properties. A minimum angle of internal friction of 25 degrees and a maximum amount of silt and clay sizes of 15 percent are used. The exclusion of greater amounts of cohesive type materials is reasoned for several purposes, not the least of which is the lack of good field experience on the long term effects of soil-strip load transfer.

In a Reinforced Earth structure, the basic design consists of several considerations, namely (a) earth; (b) reinforcements; (c) foundation stability and settlement; (d) internal and external drainage. Principally, the fill used within the Reinforced Earth mass is confined top and bottom by a layered system of reinforcing strips. Any lateral strain that takes place is that which is taken by the reinforcement, providing that the frictional resistance mobilized between the strips and the soil is sufficiently large. Hence, the need for a soil exhibiting essentially frictional characteristics.

As in other composite materials whose moduli vary considerably, the Young's modulus of the strip is much higher than the deformation modulus of the soil. Therefore, any strains are controlled by the stiffer metallic material. Essentially, such minor strains assure no change in the state of stress of the soil from the as-constructed to finally loaded condition. Each

reinforced mass is designed based on the design tensile strength of the reinforcing strips.

As you know, soils are not able to resist tension. We now have a means of using normally available soils reinforced with internal, thin metal strips to resist any imposed load. Since the strips can be placed in any direction, the anisotropic properties of the reinforced mass can be easily modified to fit the design loading conditions.

Let us take a look at the reinforcing strips themselves (Figure 2). To assure the internal stability of the earth structure, the interaction of the earth and the reinforcements has to be estimated. If we assure that the friction between soil grains and the metal strips is developed, the "non-slip" analysis allows us to define the forces that must be taken in the reinforcement. If we assume a finite element such that the incremental tensile force between two soil grains is  $(dF)$ , then the product of the normal force on the planes of the strip  $(2N)$  over the finite length  $(dl)$  and the earth-strip coefficient of friction must be equal to or exceed  $(dF)$ .

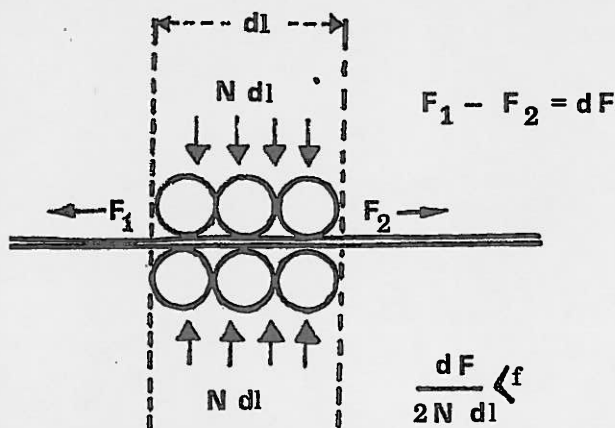


Figure 2.

Further, since reinforcements are strips of limited width with specific spacing on any plane,  $dF$  must be distributed according to the area-strip ratio  $(K)$  for any plane studied. With this ratio and a suitably applied safety factor  $f$ , the formula in Figure 2 becomes  $dF/dl < L2KNf$  and from it the number and placement of metal strips may be easily determined.

Frictional load transfer is much more positive if the faces of the metal strips are grooved or otherwise distorted. The action of the soil between strips is not fully determinable; however, the load transfer mechanism is thought to be of an arching nature.

To illustrate the design concept, let us examine a vertically faced reinforced earth wall, with horizontal strips and retaining a horizontal backfill. On every

horizontal plane, the vertical stress is equal to  $\gamma H$ . At any intermediate point the intermediate principal stress is some lesser value dependent on the coefficient of earth pressure. This stress must be resisted by the horizontal strips which in turn create shear stresses in the soil around the strips. Since a prediction of the actual state of stress within the soil is difficult, field measurements have been made to determine approximate distributions. Such measurements show the highest stress concentrations in the lower wall elements (active state) with the upper zones closer to at rest conditions.

Failure within a reinforced earth wall may take place in essentially two ways:

1. insufficient length of strips to assure safe friction transfer and
2. shear through the Reinforced Earth mass.

In practice, no Reinforced Earth structure is designed with its width less than 0.8 times its height. This is felt to offer a conservative safety factor against improper embedment depth.

Shearing through the earth-strip mass can be analyzed by normal earth pressure theories, the tensile force to be resisted by the strip reinforcements being equated to the available steel area at any level in the wall.

Actual field measurements show conclusively that the greatest tensile forces occur along the strip near the face of the wall and decrease on a near-linear distribution toward the free end (Figure 3).

### DESIGN EXAMPLE

The following wall configuration is to be designed for a level backfill loading as shown in Figure 4, where:

- $L = 0.8H$  (assumed),
- $e = (1/3)H$ ,
- $\gamma = 120$  PCF,
- $\phi = 30^\circ$ ,
- $W =$  weight of Reinforced Earth wall, and
- $P =$  active force due to backfill.

For equilibrium the following equations must be met:

1. Sum of vertical force = 0

$$W - L(a + b)/2 = 0;$$

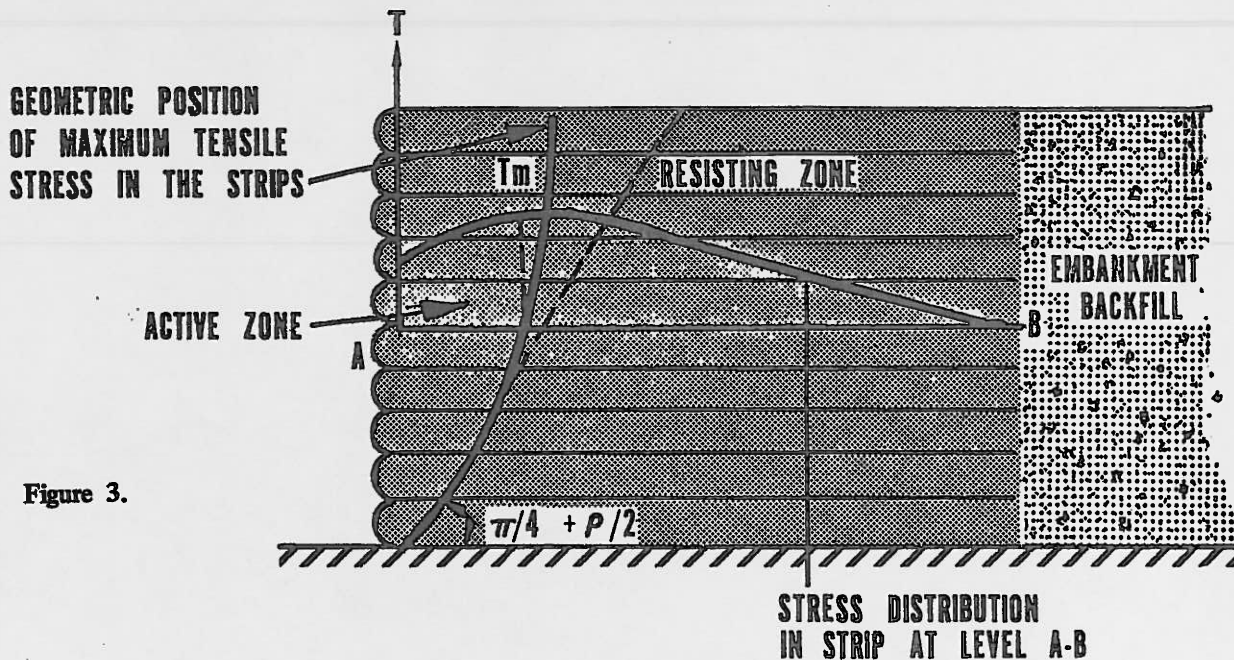
2. Sum of moments about base center = 0

$$Pe - (b - a)L^2/12 = 0.$$

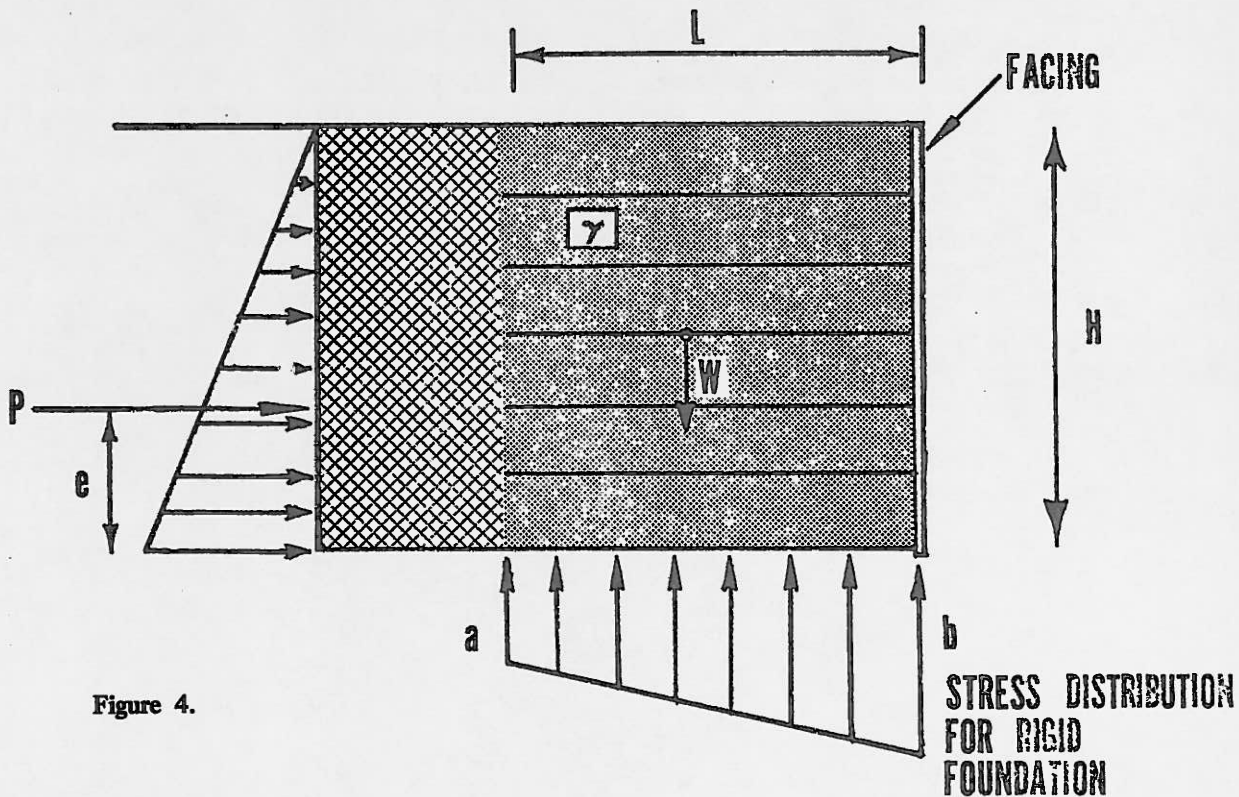
Stresses in the foundation:

- $a = 1.7$  kips/ft<sup>2</sup> and
- $b = 5.5$  kips/ft<sup>2</sup>.

# STRESS DISTRIBUTION IN REINFORCED WALL



# FORCE SYSTEM ON REINFORCED WALL



The traction force in the strip-bed close to the face is given by:

$$T_i = i K_a \gamma (1 + K_a) (\Delta H)^2,$$

where:  $i$  = number of strip beds counting from the top,

$\Delta H$  = spacing of strip beds, and

$K_a$  = coefficient of active pressure.

For this problem with a strip-bed spacing of 10" (.83'), the traction in the lower bed becomes:

$$T = 36 \times .33 \times .12 \times (1 + .33) \times (.83)^2 \\ = 1.33 \text{ kips/ft.}$$

### SHEARING STRESSES

Shearing stresses due to earth pressure behind the wall and external loads above or inclined to the wall must also be considered in design. If we assume that shear developed along any horizontal plane due to external loadings must be resisted by friction with the soil between strip layers, then:

$$\text{S.F.} = 2\gamma H_n L \tan \phi / K_a \gamma H_n^2$$

or

$$\text{S.F.} = (2 \tan \phi / K_a) \times (L / H_n).$$

Thus the least safety factor would be at the lowest level in the wall, or where  $H_n = H$ . For our wall,

$$\text{S.F.} = (2 \times 0.5777 / 0.33) \times (24 / 30) = 2.8.$$

Turning next to the external stability of the Reinforced Earth structure, design analyses are carried out similar to normal soil mechanics procedures. All work must be evaluated for bearing capacity, sliding along the base, and general stability of the Reinforced Earth mass.

Settlement of completed structures is also an important consideration; however, the flexibility of the Reinforced Earth structure allows much more relative differential movements than a rigid structure can, without risk to the integrity of the design.

One very important consideration in Reinforced Earth design is judicious consideration of drainage of the designed structure. Hydrostatic forces are not normally part of the force system included in the design analysis due to the use of free draining materials within the structure backfill. However, to assure positive drainage around the Reinforced Earth mass, careful consideration should always be given to base drainage

below the structure, and the possibility of weep holes in the skin element face.

### CONSTRUCTION

Construction of Reinforced Earth structures is quite simple and requires no special technical knowledge above proper earthwork placement procedures. The most difficult element is foundation preparation where excavations are required to provide adequate resistance to sliding in talus slopes or landslide areas.

To illustrate these points, several case histories will be discussed. The first one is of a Reinforced Earth wall built on Route 39 in the Angeles National Forest in California. This is the first highway project in the United States on which this technique was used.

In the winter of 1969 this route was closed by a massive avalanche-type slide which covered the roadway with some 35 feet of debris. The slide extended approximately 800 feet above the roadway elevation on a slope of approximately 1-1/2:1 and it was not feasible to reopen the highway using the old highway alignment. Since the decision was made that this section of the highway should not be abandoned, three alternates were suggested to provide a crossing of the canyon: a structure, a buttress wall, and a Reinforced Earth wall.

Because of the physical setting of the projects, these structures would have to be designed for the following criteria:

- a canyon width of 350 feet,
- earthquake forces, since the project is only 12 miles from the San Andreas fault,
- avalanche forces or accumulation of debris behind any wall type structure, because of the strong possibility that other avalanches would develop above the proposed crossing, and
- poor foundation material because of the thickness of the slide debris in the canyon.

After a thorough subsurface investigation of the site and after a visit by Mr. Henri Vidal, the French engineer who conceived the Reinforced Earth concept, estimates were formulated for the three alternates. The Reinforced Earth wall alternate offered a savings of \$1 million compared to the next lowest estimate.

A cross section of the final design is shown in Figure 5. The Reinforced Earth wall rests on an embankment with a side slope of 1.75:1, which itself is buttressed at the toe approximately 150 feet below the wall. Because it was not practical to remove and recompact a 15 to 20 foot layer of the slide debris, the stability of the wall and the embankment was dependent on the stability of the talus debris (Figure 6). An extensive drainage system was included to drain

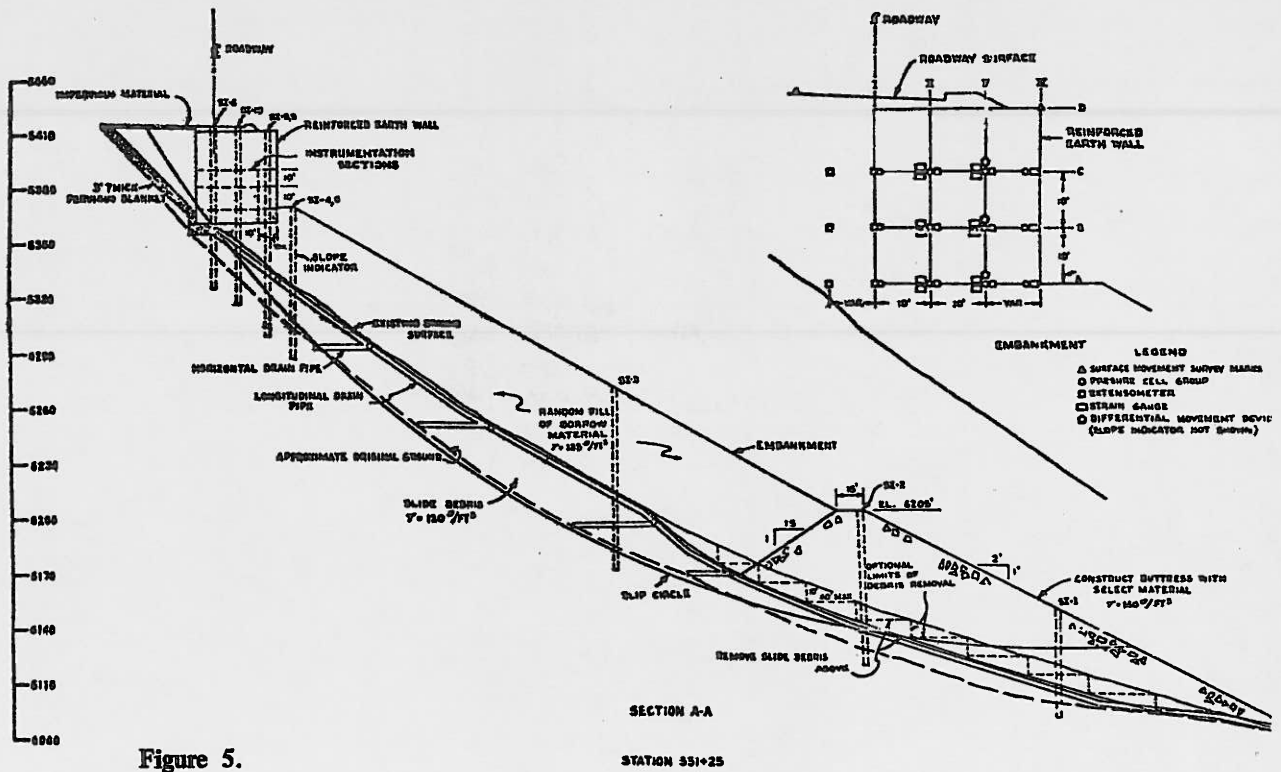


Figure 5.

CALIFORNIA RTE. 39

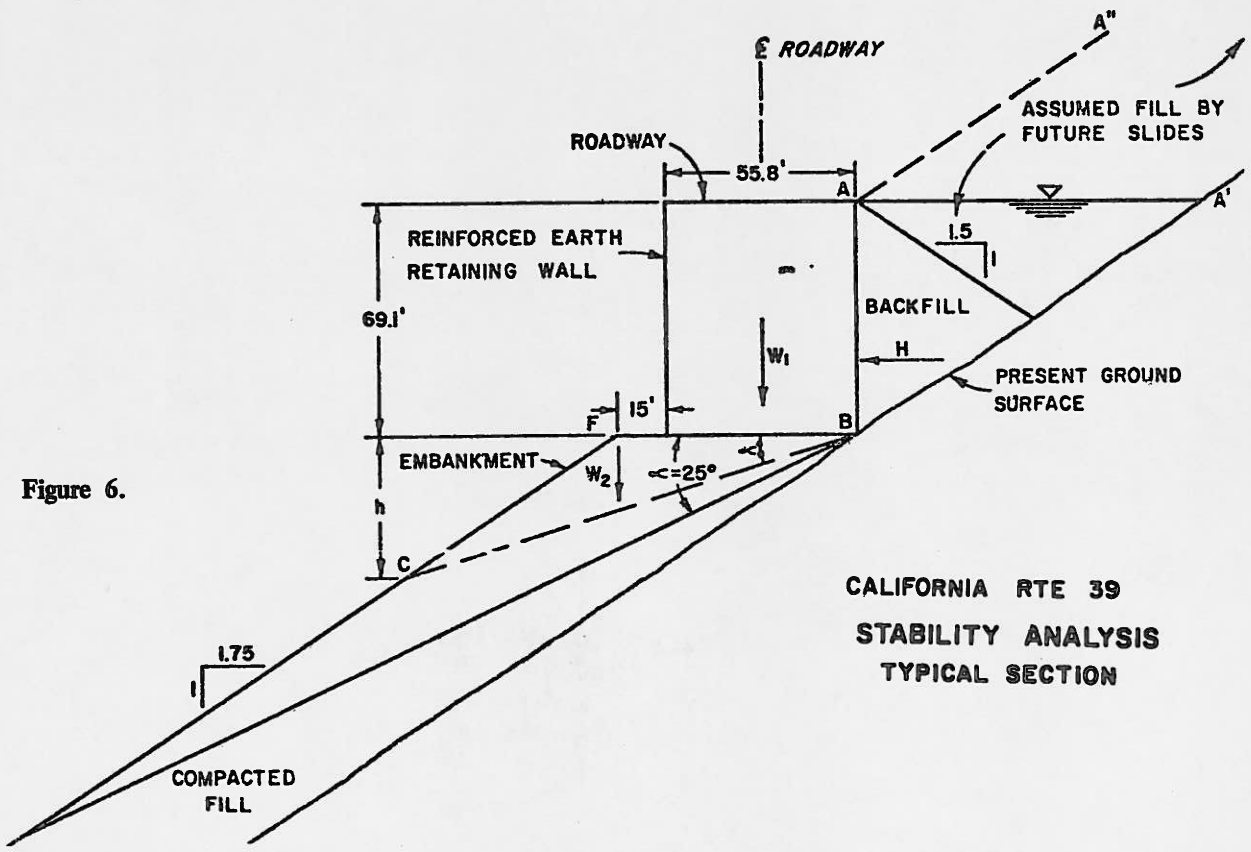


Figure 6.

CALIFORNIA RTE 39  
STABILITY ANALYSIS  
TYPICAL SECTION

water that might collect in the debris. As a precautionary measure, the stability of the wall was also designed with a saturated backfill to assure stability should rainfall be extremely heavy.

To allow for the possibility of further slides above the wall, an area was left behind the wall to catch any debris. The stability analyses included a condition in which this area was debris-filled with a slope parallel to the natural slope.

In order to verify the design assumptions, the project was heavily instrumented to monitor movements and stresses that occurred during and after construction of the wall. The cross section and plan shown in Figures 5 and 7 respectively, indicate the location of this instrumentation. Subsequent visits to the project site have shown that the wall is serving satisfactorily and that no structural damage has occurred to the wall under the approximate 3 feet of differential settlement that has occurred.

The second case history is a different type of application of Reinforced Earth presently being used near Philadelphia, Pennsylvania. The proposed alignment of SR 202 near Norristown crosses an area which is known to be active due to sink hole formation. It was determined that a bridging structure was necessary through this area in order to support the roadway. The structure would have to be designed to carry the roadway should a 50 foot-diameter sink hole develop. Roadway profile through the section involved varied from shallow cuts to a maximum fill height of 20 feet.

Both a reinforced concrete slab and a Reinforced Earth slab were proposed as alternates. Preliminary estimates indicated the Reinforced Earth slab to be some \$500,000 less than the concrete slab.

Since all the bidders on this project chose the Reinforced Earth slab alternate, no actual cost comparisons are possible. The designed Reinforced Earth slab consists of a 3 foot layer of granular backfill (sub-base material) in which layers of reinforcing strips have been placed. These strips are placed longitudinally and transversally in alternating layers within the slab (see Figures 8 and 9). The strips, which are 80 mm x 3 mm (3.15" x .118"), are placed in both directions on a 5 inch spacing, center to center. The width of the slab varies from 80 feet to 160 feet and is continuous for the 1200 foot problem area on this project.

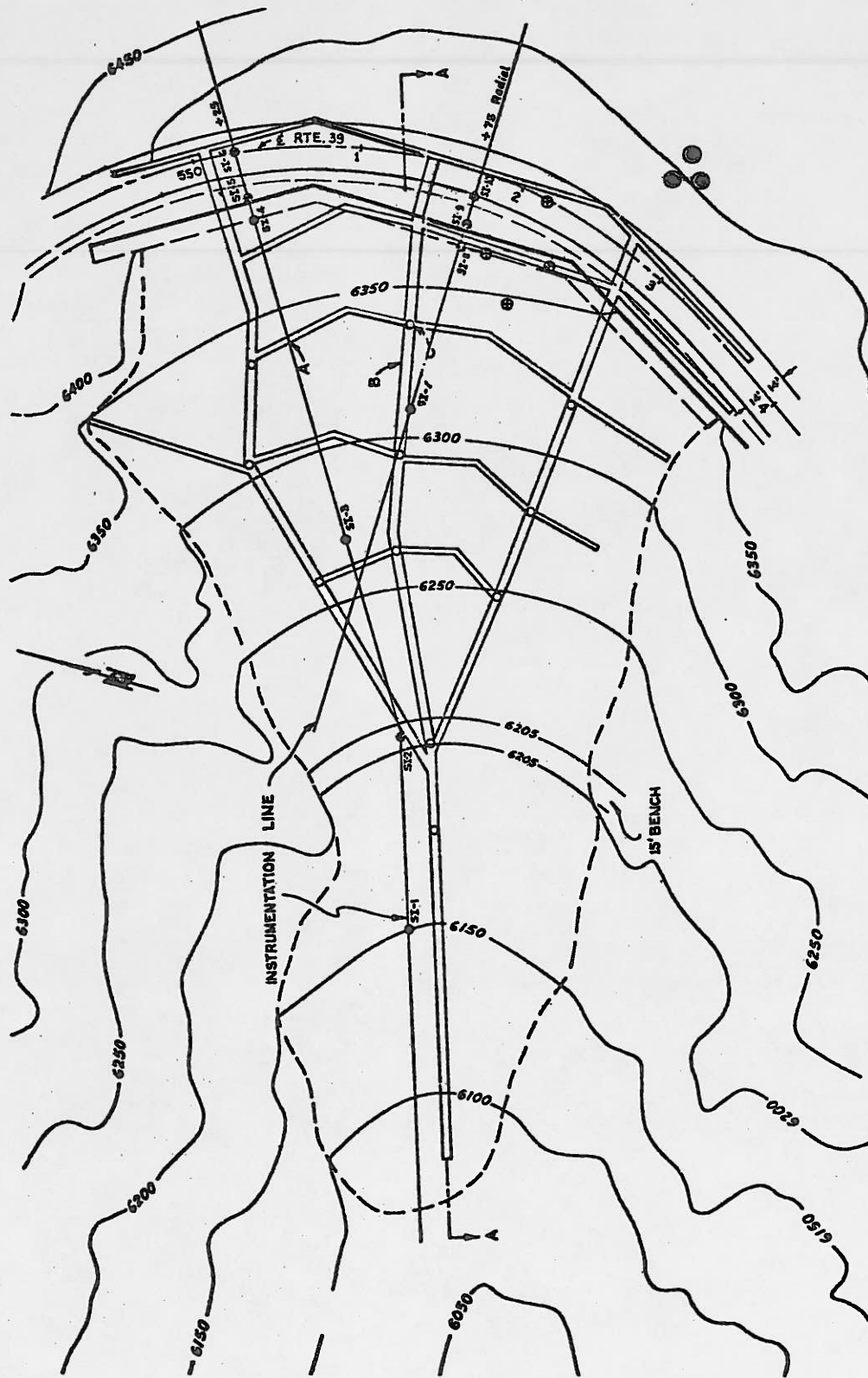
Since the principle of Reinforced Earth is based on soil-strip friction, a certain minimum overburden on the slab is required to develop the resistance necessary in the strips, should a sink hole develop. To confirm the design assumptions, the Reinforced Earth slab will be heavily instrumented during construction to record displacements and stresses within the reinforcing strips.

Other applications of Reinforced Earth exist and presently are under study in France by the parent company, Terre Armee and the central laboratory des Ponts et Chaussées. One of these is the effect of cohesive soils on the strip-earth strength if this type of material were used as a backfill material. Another is the structural life of the reinforcing strips tested under many different corrosive environments. Different steels and aluminum alloys are also under test, since their corrosion resistance will affect the life of the structure.

Basically for design, the life of Reinforced Earth structures is broken down into four categories: Temporary structures with a design life of approximately 5 years or "permanent" structures with a design life of 50-60 years with each one of these two categories divided into "dry" structures or partially submerged structures. This last category involves special design and construction procedures if the structure is to be built in the wet.

The Reinforced Earth Company has experimented with different construction procedures, with the earlier techniques requiring underwater labor by scuba divers. Presently, the technique consists of building sections of the wall in the dry with a rigid frame holding the facing and the strips in place. Each unit of wall is then lowered into place with a coupling arrangement between the units. The wall is then hydraulically backfilled with a suitable material. A typical section of a completed wall is shown in Figure 10.

To date, no structure of this type has been constructed in the United States, but one is designed for a project in Idaho. In summary, I have tried to illustrate through these few examples that, from an engineering standpoint, Reinforced Earth is a sound and versatile construction material which can be used competitively in the construction field. I think this should also illustrate that there is room in this field for innovative construction methods and materials in order that we can get a better return for our construction dollar.



**PLAN**

**LEGEND**

- A-6" ▯ TRANSVERSE DRAIN PIPES
- B-16" ▯ LONGITUDINAL DRAIN PIPES
- C-8" ▯ HORIZONTAL DRAIN PIPES
- ALL PIPES PERFORATED ON 1/2 CIRCLE
- SI - SLOPE INDICATOR
- ⊙ - APPROXIMATE LOCATIONS OF FIELD DENSITY TESTS
- ⊙ - APPROXIMATE LOCATIONS OF FIELD PERMEABILITY TESTS

Figure 7.



# SLAB APPLICATION OF REINFORCED EARTH

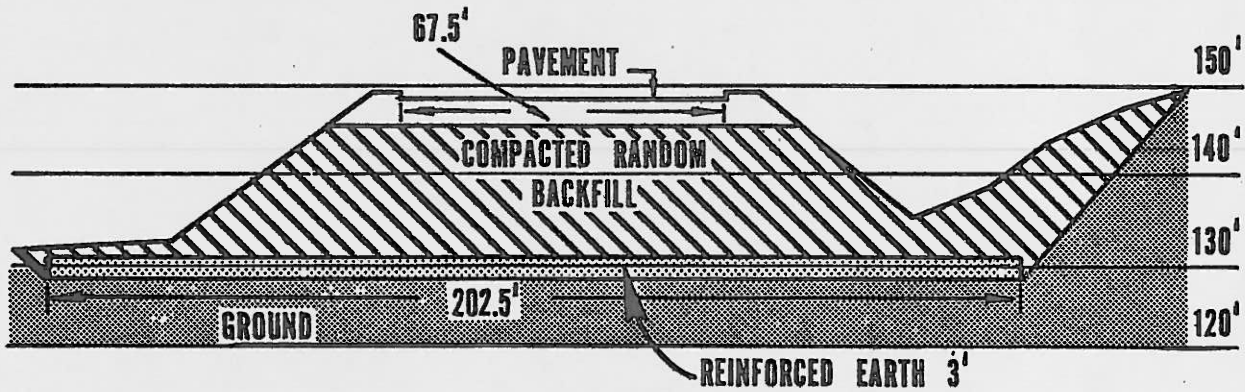


Figure 8.

## DETAIL OF REINFORCED EARTH SLAB

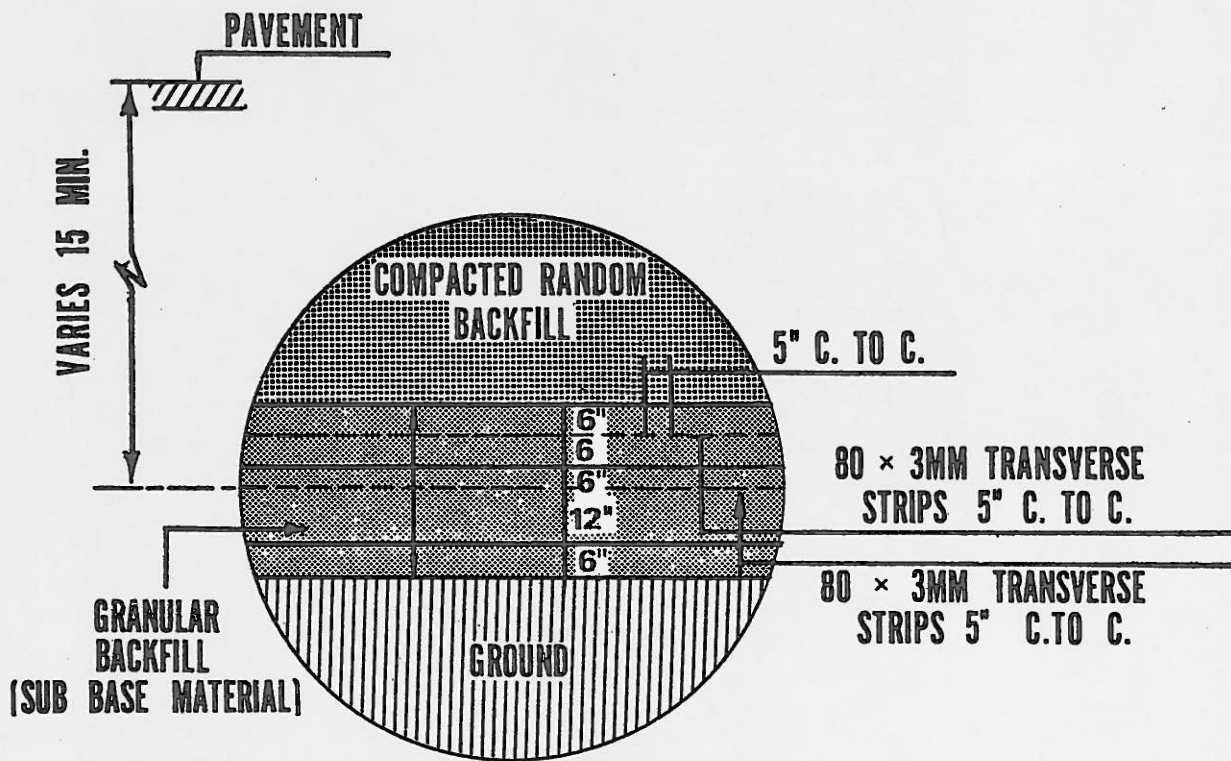
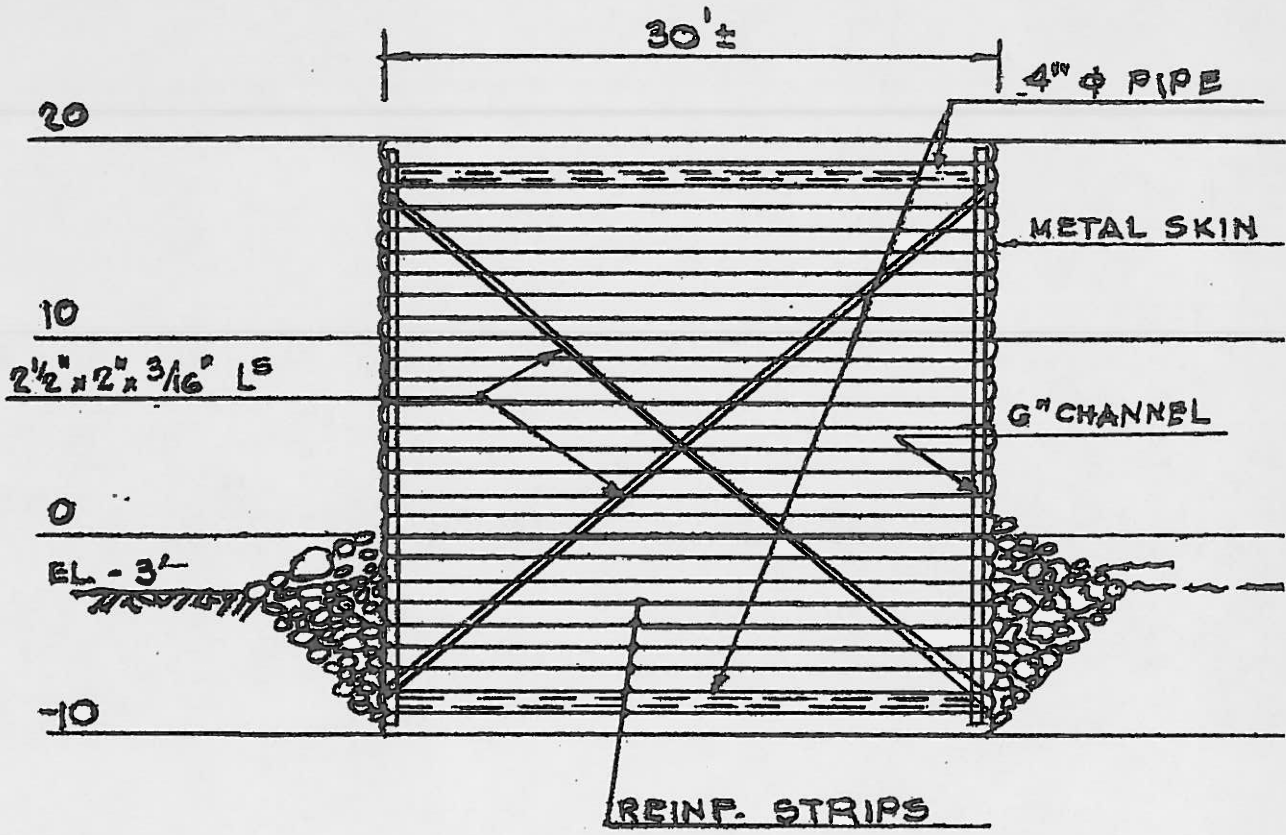


Figure 9.



TYPICAL SECTION

Figure 10.

# APPLICATIONS OF THE FINITE ELEMENT METHOD TO GEOTECHNICS AND TRANSPORTATION ENGINEERING

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## INTRODUCTION

In the past decade, great advances have been made in the applications of the finite element method to geotechnics and transportation engineering. Many problems, which were considered intractable a few years ago, can now be solved consistently by the finite element method with a minimum of over-simplifying idealizations. This ability together with the accumulating evidence of documented applications has given the method a credibility unaccrued by any other analytical technique. The finite element method has gradually become a set of magic words and was mentioned frequently by engineers for solving difficult problems in geotechnics and transportation engineering.

The original concept of the finite element method was developed in 1956 (16) for analyzing the stresses and deflections in complex structures. The method was later related to the variational principle and allowed its almost unlimited extension to other fields than that of stress analysis. The first application of the method to geotechnics was reported in 1962 (3) for analyzing a concrete dam on rock foundation. The potential of the method in flexible pavement design was recognized in 1967 during the Second International Conference on the Structure Design of Asphalt Pavements (17), and its application to the analysis of an in-service pavement was reported in 1968 (5). Although the classical problem of plates on elastic foundations was solved by the finite element method in 1965 (2), its application to the analysis of rigid pavements was made only recently (12). In 1972, a symposium on the applications of the finite element method in geotechnical engineering was held at Vicksburg, Mississippi, in which 35 papers including six state-of-the-art reports were presented (4).

It is interesting to note that, in spite of the great potential of the finite element method, its use has so far been limited to universities, research agencies, or large engineering firms. For most practicing engineers, very little is known about the method. One factor which makes the method inaccessible to most engineers is the enormous amount of computer time required for solving a realistic problem. The rapid growth of the finite element method was due in part to the availability of "free" computer time on university campuses. It was

reported (14) that a civil engineering department in a major university was allocated an amount of computer time, the cost of which at commercial rates would approach the cost of the entire departmental budget, and that a graduate student working three years on his doctorate had used computer time which at commercial rates would have cost \$100,000. With the advent of more powerful computers and more effective finite element programs, it is believed that the cost of finite element analysis can be greatly reduced and that in the near future the method can be made available to most practicing engineers.

The purposes of this paper are twofold: (a) to describe the potential uses of the finite element method in a way which can be understood by practicing engineers who are unfamiliar with the method, but who would like to know how it can be applied to practical problems in geotechnics and transportation engineering, and (b) to describe several research projects undertaken by the author during the past few years which can illustrate the direct application of the finite element method.

## WHAT IS THE FINITE ELEMENT METHOD?

The finite element method is a numerical technique for solving engineering problems. Instead of considering the structure or continuum as a whole, the behavior of each constituent part is studied and then combined to predict the behavior of the whole. This concept has been used for a long time in structural analysis. The well-known Hardy-Cross method of moment distribution was based on this concept.

Fig. 1 shows a simple structure consisting of three pin-connected bars. The problem now on hand is to determine the displacement at joint 2 under the action of a force  $P$ , which can be resolved into a horizontal component,  $P_x$ , and a vertical component,  $P_y$ . In the finite element method, each bar is considered as a one dimensional element with two nodes  $i$  and  $j$ , as shown in Fig. 2. The property of each element is characterized by an element stiffness matrix,  $[K]$ , which relates nodal forces,  $\{F\}$ , to nodal displacements,  $\{\delta\}$ , or

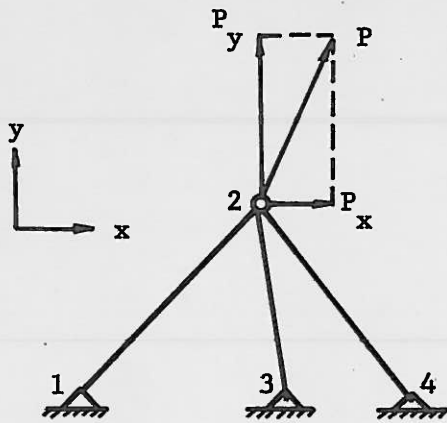


Fig. 1. A Simple Structure.

$$[K] \{\delta\} = \{F\}. \quad (1)$$

Standard procedures, based on the compatibility of displacements, are available to generate the stiffness matrix of one, two, and three dimensional finite elements. As long as the material properties and the dimensions of an element are given, its stiffness matrix can be determined readily. For the one dimensional element shown in Fig. 2, each node has two forces, a horizontal force  $U$  and a vertical force  $V$ , and two displacements, a horizontal displacement  $u$  and a vertical displacement  $v$ . Since each element has two nodes and each node has two forces and two corresponding displacements, the dimension of the stiffness matrix is 4 by 4, or

$$\begin{bmatrix} UU_{ii} & UV_{ii} & UU_{ij} & UV_{ij} \\ VU_{ii} & VV_{ii} & VU_{ij} & VV_{ij} \\ UU_{ji} & UV_{ji} & UU_{jj} & UV_{jj} \\ VU_{ji} & VV_{ji} & VU_{jj} & VV_{jj} \end{bmatrix} \begin{pmatrix} u_i \\ v_i \\ u_j \\ v_j \end{pmatrix} = \begin{pmatrix} U_i \\ V_i \\ U_j \\ V_j \end{pmatrix} \quad (2)$$

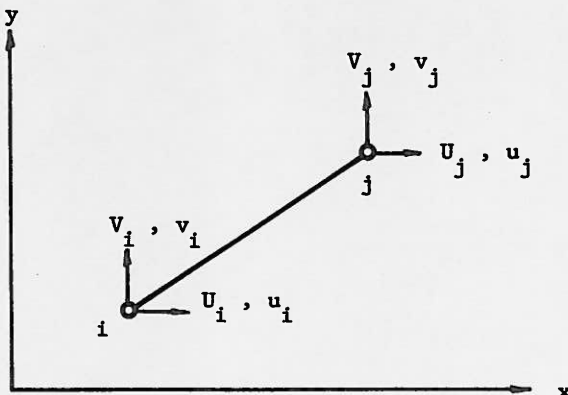


Fig. 2. One Dimensional Element.

in which  $UU_{ii}$ , etc., are coefficients of the stiffness matrix. It is hoped that the representation of the element stiffness in matrix forms, as shown in Eqs. 1 and 2, does not cause confusion to those who are not familiar with matrix notations. In fact, Eq. 2 is nothing but a condensed representation of four equations, each for determining one of the nodal forces. For instance, the first equation gives the horizontal force at node  $i$  by

$$U_i = (UU_{ii}) u_i + (UV_{ii}) v_i + (UU_{ij}) u_j + (UV_{ij}) v_j. \quad (3)$$

Eq. 3 shows the effect of the four displacements, i.e.,  $u_i$ ,  $v_i$ ,  $u_j$ , and  $v_j$ , on  $U_i$ . A nodal displacement,  $u_j$  or  $v_j$ , induces a nodal force,  $U_i$ , through a stiffness coefficient,  $UU_{ij}$  or  $UV_{ij}$ , which is defined as a nodal force due to a unit nodal displacement. For most problems in geotechnics and transportation engineering, the nodal displacements are considered as unknown quantities to be determined by solving a set of simultaneous equations. If the stiffness matrix for the three bars in Fig. 1 is determined, the horizontal and vertical forces at joint 2 due to each bar can be written in terms of its nodal displacements. Since the sum of the three vertical forces, one due to each bar, must equal to the external force  $P_y$ , one equation involving the unknown nodal displacements is obtained. The other equation can be obtained by setting the sum of the three horizontal forces to  $P_x$ . It can be seen that two equations based on the equilibrium of forces can always be set up at each joint, where there are two unknown displacements. When a structure is composed of  $n$  joints, a total of  $2n$  equations can be obtained to solve the  $2n$  nodal displacements. For the structures shown in Fig. 1, only two equations are needed to determine the vertical and horizontal displacements at joint 2, because the displacements at nodes 1, 3, and 4 are known to be zero.

The finite element technique described above can be easily extended to two and three dimensional analysis. The basic element for the two dimensional analysis is a triangle, as shown in Fig. 3. Since each triangular element has three nodes and each node has two components of forces and displacements, the dimension of the stiffness matrix is 6 by 6. The basic element for the three dimensional analysis is a tetrahedron, as shown in Fig. 4. Since each element has four nodes and each node has three components of forces and displacements, one along each of the axes, the dimension of the stiffness matrix is 12 by 12. Once the stiffness matrix for each element is found, a set of simultaneous equations, based on the equilibrium of forces at each node, can be obtained to determine the

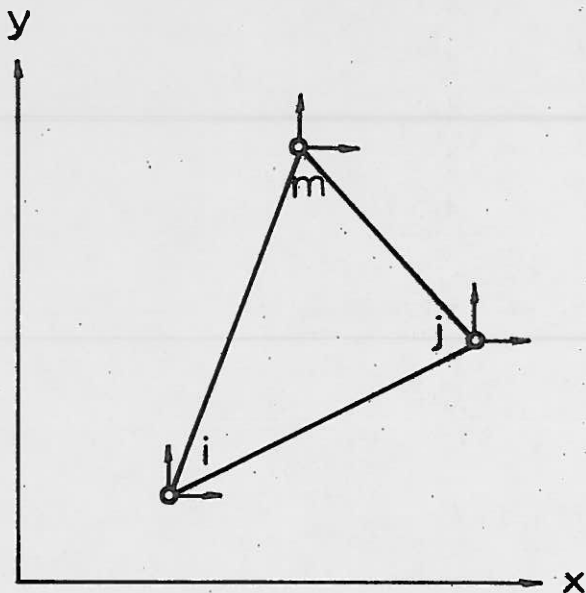


Fig. 3. Two Dimensional Triangular Element.

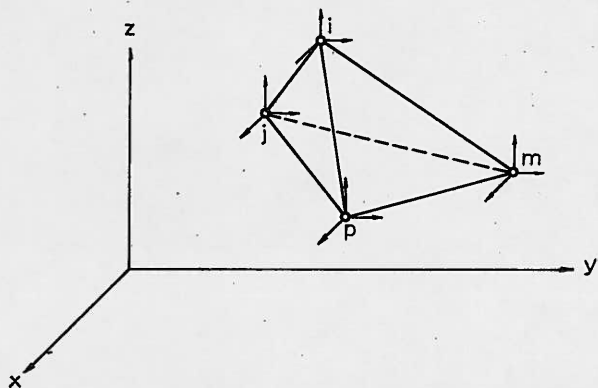


Fig. 4. Three Dimensional Tetrahedral Element.

unknown nodal displacements. If the external loads are not applied directly to the nodes but are distributed over a given volume or area, they should be replaced by a set of equivalent nodal forces by means of statics. After the nodal displacements are determined, the stresses and strains can then be found.

The above approach, based on the compatibility of displacements and the equilibrium of nodal forces, is called the direct method of formulation. An alternative approach based on the minimization of potential energy also yields the same formulation. This energy concept can be extended to a variety of nonstructural problems in which a variational principle exists. In such problems, the exact solution is defined as that which minimizes some integral of the unknown

function or of its derivatives. This integral is known as the functional of the problem. If the unknown function is defined throughout the region, element by element, in terms of the nodal values of the function as indicated by Eq. 1, the minimization of the functional will result in a set of ordinary equations equal in number to that of the unknown values of the function at the node. If the problems are of a type in which the functional is an integral of terms involving the unknown function or its derivatives in powers of zero, one, or two only, the equations resulting from the minimizing process will be of the same type as those involved in linear stiffness analysis of structures.

#### WHAT ARE THE ADVANTAGES AND DISADVANTAGES OF THE METHOD?

The finite element method can be used to solve many complex problems in geotechnics and transportation engineering which cannot be solved by other analytical means. The major advantages of the finite element lie in its simplicity in handling nonhomogeneous, discontinuous, and nonlinear media with irregular shape and cross section, which are typical of natural geologic and soil formations. When a medium is divided into a number of finite elements, the stiffness or property of each element can be changed at will to simulate actual nonhomogeneous conditions. If a discontinuity or plane of weakness exists, which cannot take tension perpendicular to that plane, an iterative scheme can be used to eliminate the forces at those nodes on the plane where tension is indicated (1). The two and three dimensional elements shown in Figs. 3 and 4 can be used to approximate any boundaries consisting of a series of straight lines or planes. Curvilinear boundaries can be approximated by isoparametric elements (17).

The most striking advantage of the finite element method is its ability in analyzing nonlinear problems. It is well known that soils and rocks are nonlinear with a typical stress-strain relationship shown in Fig. 5. If the material property is characterized by the tangent modulus,  $E$ , it can be seen from Fig. 5 that the tangent modulus is stress dependent, being greater at lower levels of stress and smaller at higher levels of stress. For such a nonlinear material, a question immediately arises: "What should be the tangent modulus to be used in the finite element analysis?" This problem can be solved conveniently by the incremental loading method. Starting from zero, the load is divided into increments. The initial tangent modulus,  $E_1$ , is used for the first load increment, and the stress,  $\sigma_1$ , in each element is determined by assuming the medium as linear with a

modulus  $E_1$ . Then go to the second load increment. A tangent modulus  $E_2$ , based on the stress  $\sigma_1$ , is determined for each element from the stress-strain curve. The stress due to the second incremental load is computed by assuming the medium as linear with a modulus  $E_2$ . The stress thus computed is added to  $\sigma_1$  to obtain the total stress  $\sigma_2$  at the end of the second load increment. From  $\sigma_2$ ,  $E_3$  can be determined. The process is repeated until all increments are considered.

The incremental loading method is most practical for analyzing the stresses, strains, and displacements in a soil or rock medium under a sequence of construction steps. The increments may involve excavation, fill placement, changes in water pressure, and applications of loads. The stresses in the ground before construction are required to define the initial conditions. The strains and displacements before construction are usually taken as zero. The construction is divided into a series of steps. Increments of stress and displacement for each step are calculated and added to the values for the previous step, thereby providing information on stresses, strains, and displacements for each stage of construction. Because the actual sequence of events is modeled in the analysis, the calculated stresses, strains, or displacements are of the same nature as those actually observed, and direct comparisons between measured and observed values are therefore possible.

Although the finite element method can be used to solve various problems in engineering practice, it also suffers from several limitations. The application of the method requires a relatively large amount of computer memory and time and is therefore quite expensive. For certain classes of problems, the method, by its inherent nature of requiring relatively large computation for element data, may become less efficient than the finite difference method. As in any other numerical methods, the finite element method can only provide approximate solutions, and large errors may result if the medium is not divided properly. The solution of the large number of simultaneous equations is time consuming, and round-off errors may cause inaccuracies in part of the solution, especially when the values of interest are comparatively small. Because of the great complexities in geotechnical problems, it is difficult to develop a general computer program which can serve all purposes. Consequently, revisions and modifications are constantly needed, and a great deal of time and effort is required. The principle of the finite element method can be understood very easily, but finite element computer programs are so complex that several weeks or months are required to learn how the program works. To avoid the danger of possible misuse, an investment in training should be considered a necessary first cost by any group interested in using the method.

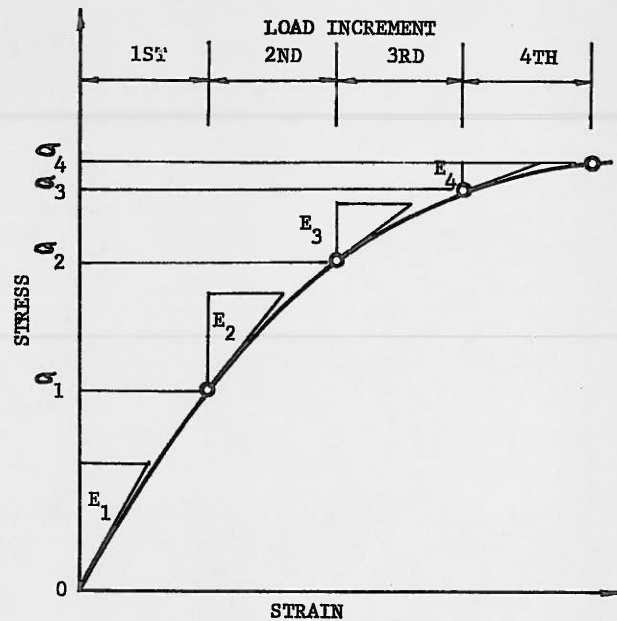


Fig. 5. Nonlinear Stress-Strain Relationship.

To use the finite element method, the constitutive equations describing the material properties must be prescribed. It is generally agreed that the development of the finite element technique is much more advanced than that of the constitutive equations. If the material properties cannot be defined properly, it is senseless to make an expensive finite element analysis, because the outcome of the analysis depends completely on the material properties assumed. To exploit fully the power of the finite element method, significant effort must be directed toward the development of suitable constitutive laws and the evaluation of realistic material parameters.

It must be emphasized that the main advantage of the finite element method lies in its capacity of handling complex problems. For simple problems, conventional methods based on closed form solutions may be more preferred. For instance, it is a waste of time and effort to use the finite element method for computing the one dimensional consolidation of a homogeneous soil, because the well-known Terzaghi theory would yield quicker and more economical solutions. However, there are exceptions that favor the finite element method over the conventional methods. When a complete picture of the distribution of stresses and displacements in a pavement, which is considered as a linear elastic multilayer system, is desired, the finite element method is more efficient because it can compute the stresses and displacements at many points all at the same time, with very little additional effort if more points are computed. When the stresses and displacements at only one or two points are desired, the well known Burmister theory is more suitable.

## WHAT USES CAN BE MADE OF THE METHOD?

The finite element method can be used to solve two types of problems in geotechnics and transportation engineering, viz. structural type and nonstructural type. The structural problems involve the determination of stresses, strains, and displacements in dams, embankments, open excavations, slopes, pavements, foundations, retaining walls, braced excavations, and underground structures. The nonstructural problems involve the analysis of consolidation, heat conduction, frost penetration, seepage, fluid flow, and possibly traffic flow. The six state-of-the-art reports in the Vicksburg conference have cited hundreds of references related to these applications. Because of space limitations, no attempt will be made here to cite these references. Instead, only a general description on the potential applications of the method will be made.

### Design and Analysis

The design of a structure requires information on the stresses, strains, and displacements in the structure. The thickness of a rigid pavement is governed by the critical tensile stress in the concrete slab. In installing wells for water supply, the drawdown at the wells must be estimated, so that an effective pumping system can be designed. Even under complex field conditions, the finite element method can provide direct solutions for design purposes. The method is particularly useful for parametric studies and for problems in which a considerable amount of data does not exist. It can be used to test one system against the other, so that a most economical design can be obtained.

### Evaluation of Existing Structures

A major crack may occur in a structure such as a concrete dam. It is necessary to determine whether this cracking is a normal phenomenon due to the presence of tensile stresses, or it should be a cause for concern because something unexpected has taken place. The finite element method can be used to compute these stresses. In case the crack is caused by high tensile stresses, the effect of the crack on the redistribution of stresses can be evaluated, and the integrity of the structure can be checked.

### Post-Failure Analysis

The Kimola Canal in Finland can serve as an example (6). The canal was excavated beginning in 1962 and completed in 1965. About nine months after completion, a deep-seated slope failure took place. Because conventional stability analyses based on vane shear strengths indicated that the slope had an adequate

margin of safety, a finite element analysis was performed to obtain a better understanding of the failure mechanism. Results of the analysis showed that there existed a curved zone of large shear strains located very close to the failure surface. It was therefore hypothesized that the failure was due to the weakening of the clay as a result of the large shear deformations.

### Monitoring and Control of Construction

During the construction of large and unconventional structures, it is generally desirable to monitor the movements of the structure and its foundation. If large movements are measured, it is necessary to determine whether the movements are anticipated or there is danger of catastrophic failure. The finite element method, due to its capability of taking the sequential construction into account, can be used to calculate the movements theoretically. If the calculated and observed movements agree reasonably, there is no cause for alarm. Otherwise, the cause of the discrepancy should be investigated, and proper precautions be taken.

### Location of Instrumentation

When a field project requires monitoring and instrumentation, it is necessary to predetermine the location of the instruments. A finite element analysis can be made to estimate the possible amount of measurements at various points so that instruments can be installed at locations of utmost interest. It is certainly a waste of time and money to place an instrument at a location where the measurements are too small to be registered by the instrument.

### Analysis of Laboratory Specimens

To use the finite element method, the parameters describing the material properties must be defined. These parameters can usually be determined from laboratory test specimens. However, laboratory specimens are sometimes subjected to boundary conditions quite different from the simplifying conditions originally assumed. For instance, the stress distribution over the cross section of a specimen in the unconfined or triaxial compression test is not uniform due to the restraining effect of the loading platens. By incorporating the actual boundary conditions in a finite element analysis, the effect of the boundary can be properly evaluated, and the real properties of the material be defined.

### Field Evaluation of Material Parameters

Because of the difficulty in obtaining representative samples, large scale field tests are sometimes conducted to determine the material parameters of a geologic

formation. These parameters can be tentatively assumed, and a finite element analysis is made to obtain theoretical solutions. A comparison between the theoretical solutions and the experimental measurements will show whether the assumed parameters are reasonable. If not, new values of the parameters will be assumed until the calculated and the observed values are in agreement. Once the material parameters are determined, they can be used with the finite element method to predict the behavior of the formation under other loading and boundary conditions.

#### WHAT FINITE ELEMENT COMPUTER PROGRAMS ARE AVAILABLE?

A number of general purpose systems based on the finite element method have been developed, particularly for problems in structural and continuum mechanics. Some of these systems are: (a) ASKA (Automatic System for Kinematic Analysis) by Institute of Statics and Dynamics, West Germany, which is a powerful general system including about 42 different finite elements; (b) DAISY by Lockheed Missile and Space Co.; (c) ELAS by Jet Propulsion Laboratory for the equilibrium of linear structures; (d) ICES-STRUDL II (Integrated Civil Engineering - Structure Design Language) by Massachusetts Institute of Technology for plane stress and strain, shallow shells, three dimensional solids, and plate bending and stretching; (e) NASTRAN (NAsa STRuctural ANalysis) by National Aeronautics and Space Administration for elastic analysis of various structures; (f) SAFE (Structural Analysis by Finite Elements) by Gulf General Atomic, Inc., for plane stress or strain, axisymmetric, three dimensional, and shell problems; (g) SAMIS (Structural Analysis and Matrix Interpretive System) by Jet Propulsion Laboratory, which contains an element library including a general one dimensional element and triangular elements for membrane and bending deformations; (h) SAP (Structural Analysis Program) by University of California at Berkeley for plane stress and strain, plates, shells, axisymmetric solids, and three dimensional frames, beams, and solids.

These systems permit solutions of a large number of linear and nonlinear problems. They are versatile, efficient and economical for large and complex problems but may not be economical for small and single problems. Some of these programs have been adopted for applications in geotechnics and transportation engineering. However, most of the programs in this field have been developed by individuals for their specific use. For example, the Naval Civil Engineering Laboratory (13) has developed a computer code AFPV (Airfield

Pavements) for solving the complete state of stress in typical layered pavement systems made of both linear and nonlinear materials. In the Vicksburg conference, each author was asked to answer a questionnaire about the computer program he used. Of the 22 papers answering to the questionnaire, 12 indicated that the programs were developed by the authors themselves, 8 indicated that the programs were originally developed by someone else but later modified by the authors, and only 2 indicated that other's programs were used with no modifications. The lack of a general program for solving various problems in soil and rock mechanics was recognized in the conference, and it was proposed that the U.S. Corps of Engineers develop a unified system called GEONAP (GEOtechnical Numerical Analysis Programs) which can be made available to engineers engaged in various geotechnical works.

In the following pages, three examples on the applications of the finite element method will be briefly described. Because of the complexity of the problems involved, none can be solved by the conventional methods.

#### ANALYSIS OF NONLINEAR SOIL MEDIA (8)

Fig. 6 shows a circular footing of radius  $a$  on a nonlinear soil medium of infinite extent underlain by rock. The reason that the soil is called nonlinear is because its modulus of elasticity is not a constant but varies with the state of stress. For granular soils, a simple constitutive equation suggested by the author (7) is

$$E = E_0(1 + \beta\theta), \quad (4)$$

in which  $E$  = elastic modulus under a given stress invariant, in psi;  $E_0$  = initial elastic modulus, or the

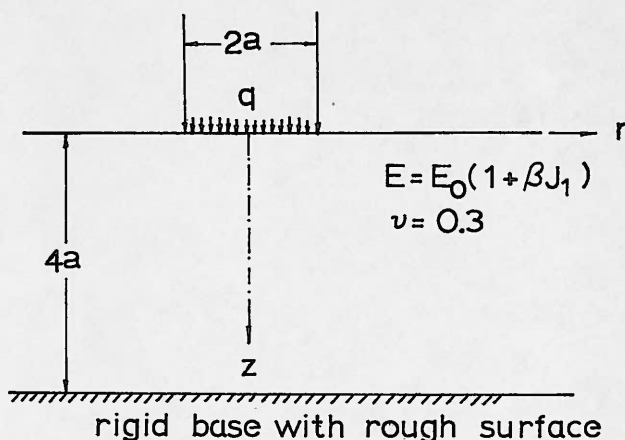


Fig. 6. Nonlinear Axisymmetric Soil Medium.



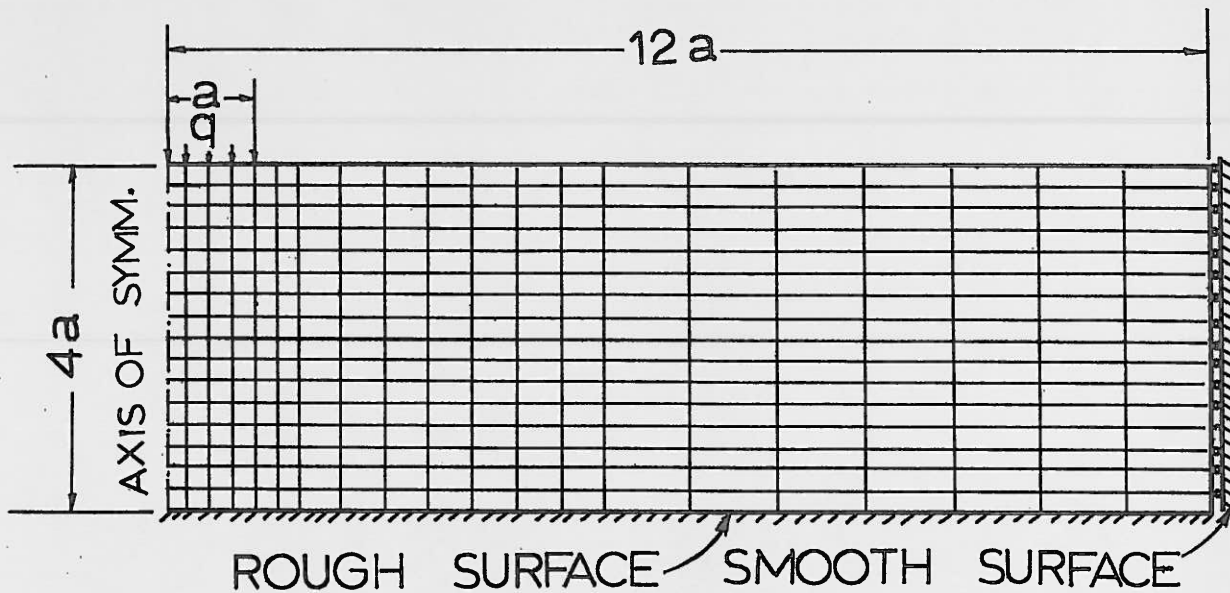


Fig. 7. Division of Soil Medium into Finite Elements.

modulus when the stress invariant is zero, in psi;  $\beta$  = a nonlinear coefficient indicating the percentage increase in elastic modulus per 1 psi increase in stress invariant, in  $(\text{psi})^{-1}$ ; and  $\theta$  = stress invariant, or the sum of the three principal stresses, in psi. Assuming that the soil medium has a thickness of  $4a$ , a Poisson's ratio of 0.3 and a nonlinear coefficient of 5, it is now necessary to determine the stresses and displacements in the medium under a uniformly distributed pressure  $q$ .

In the finite element analysis, the medium was divided into circular rings having a rectangular cross section, as shown in Fig. 7. Each rectangle was composed of four triangles. An imaginary, smooth boundary was placed at a radial distance of  $12a$  from the axis of symmetry. When the medium is linear, the stresses and displacements obtained by the finite

element method check closely with the exact solutions, indicating that the imaginary boundary has negligible effect on the analysis. The nonlinear behavior of the soil was taken into account by an iterative scheme. The first assumed the soil as linear, i.e.  $\beta = 0$ , and the stresses at the center of each rectangular element were computed. Based on these stresses, a new set of moduli was determined from Eq. 4. Using these new moduli, a new set of stresses was computed. The process was repeated until the stresses or moduli converged to a specific tolerance.

Fig. 8 shows the variation of displacements with depth. The vertical displacement at the axis of symmetry, or  $r = 0$ , and the radial displacement at the edge of the footing, or  $r = a$ , are presented. The solid curves are for a linear medium and the dotted curves for a nonlinear medium, both determined by the finite element method. The small circles are the exact solutions obtained by the stress function method. The stress distribution at  $r = 1.125a$  is presented in Fig. 9. It can be seen from Figs. 8 and 9 that in a linear medium, where exact solutions are available, the finite element method checks closely with the exact method. Of particular interest is the fact that the nonlinear behavior of soil has relatively small effect on the vertical stress. Depending upon the depth of the point in question, the vertical stress based on nonlinear theory may be greater or smaller than that based on linear theory and, at a certain depth, both theories may yield the same stress. This may explain why Boussinesq solution of vertical stress based on linear theory has been applied to soils with greater success, even though soils themselves are basically nonlinear.

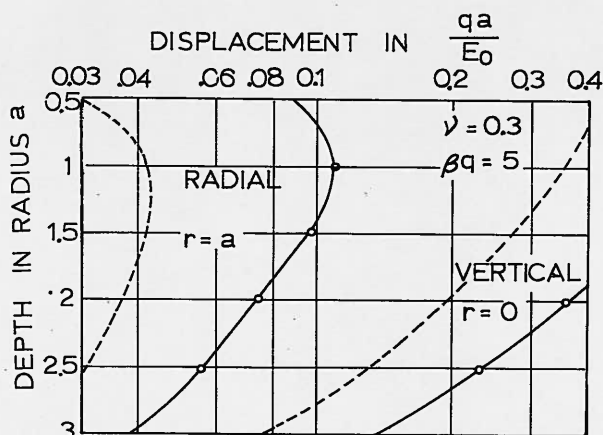


Fig. 8. Vertical and Radial Displacements.

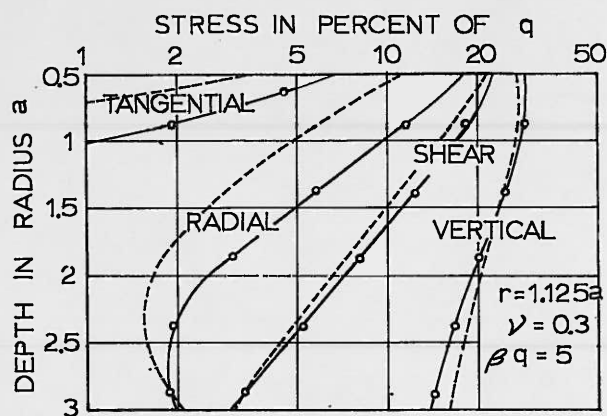


Fig. 9. Vertical, Radial, Tangential, and Shear Stresses.

### THREE DIMENSIONAL FLOW TOWARD AN ARTESIAN WELL (11)

The finite element method was applied by the author for analyzing two dimensional flow toward artesian wells (9, 10). Recently, a three dimensional finite element program was developed at the University of Kentucky (15) and applied to a hypothetical problem involving a partially penetrating infinitesimal well in a nonhomogeneous aquifer, as shown in Fig. 10. The aquifer is horizontal in position, uniform in thickness and of irregular boundaries. The transmissibility of the shaded region is assumed four times greater than that in the unshaded region, and the storage coefficient two times greater. The problem now on hand is to determine the drawdown around the well under a constant pumping rate  $Q$ .

Since the problem is axisymmetric, it cannot be analyzed by two dimensional finite elements, so three dimensional elements are employed. The elements were formed by concentric cylindrical surfaces with six-node elements surrounding the well and eight-node elements elsewhere, as shown in Fig. 11. The aquifer was divided into 380 elements with a total of 495 nodes. Starting from the well, the nodes were numbered first vertically from top to bottom, then tangentially clockwise, and finally radially outwards. The elements were numbered first tangentially clockwise, then vertically downwards, and finally radially outwards. To avoid being crowded, only the nodes and elements near the circumferential boundary of the aquifer are numbered in Fig. 10.

In the analysis, the drawdown and time were expressed as dimensionless factors as follows:

$$\text{Drawdown factor} = Ts/Q \quad (5)$$

and

$$\text{Time factor} = Tt/SL^2 \quad (6)$$

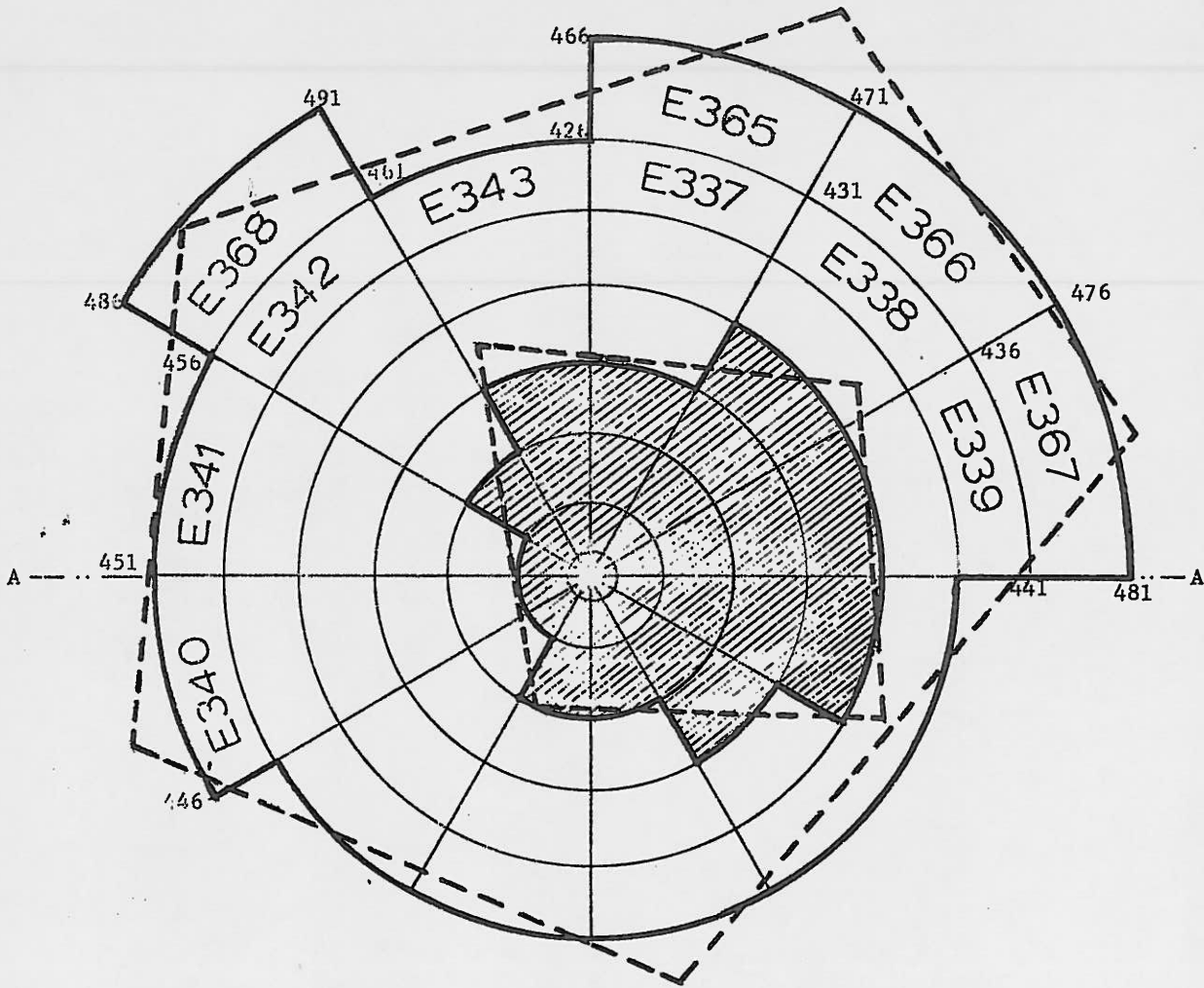
in which  $s$  = drawdown,  $T$  = transmissibility of the unshaded region,  $Q$  = discharge from the well per unit time,  $t$  = time since pumping started,  $S$  = storage coefficient of the unshaded region, and  $L$  = a unit length for measuring distances.

Fig. 12 shows the drawdown-time relationship for nodes 1, 21, and 141 located on the top impervious boundary along A-A at a distance of 0,  $2L$ , and  $12L$  from the well respectively. The circles and solid curves represent the solution for the nonhomogeneous aquifer. The rapid increase in drawdown at the final stage is due to the fact that the cone of depression has reached the circumferential boundary. Also shown but in crosses and dotted curves is the corresponding drawdown for a homogeneous aquifer. It can be seen that the drawdown for the nonhomogeneous aquifer is smaller than that for the homogeneous aquifer. This is expected because the inclusion of a more porous material with a higher permeability and a greater storage coefficient should certainly increase the supply of water and thus reduce the drawdown.

### STRESSES IN RIGID PAVEMENTS (12)

The finite element method was applied by the author for analyzing the AASHO road test rigid pavements (12). In the AASHO road test, an experiment was conducted on the non-traffic loop where the stresses due to a rapidly oscillating load at a frequency of 6 cps were measured at 15 different points distributed over a 6-ft by 6-ft area at one corner. The load was applied to the pavement through two wooden pads, each having a 11-in. by 14-in. area and spaced at 6-ft centers. The center of the outer pad was placed 1 ft from the pavement edge, thus simulating a single axleload in the traffic loops. Four loading positions were employed with the center of the load at a distance of 0.5, 2, 4, and 6 ft from the transverse joint. The loading positions, the location of points at which stresses were measured, and the finite element subdivisions are shown in Fig. 13.

It should be noted that the actual pavement consisted of a large number of slabs connected by dowels at the transverse joints and tie bars at the longitudinal joint. Because all the other slabs were quite far from the load and therefore had negligible effect on the stresses at the corner area, only two slabs were considered in the finite element analysis. It was assumed that the dowels between the two slabs could transfer shear but not moments, and that the deflections along the joint were the same for both slabs. A Young's



PLAIN VIEW

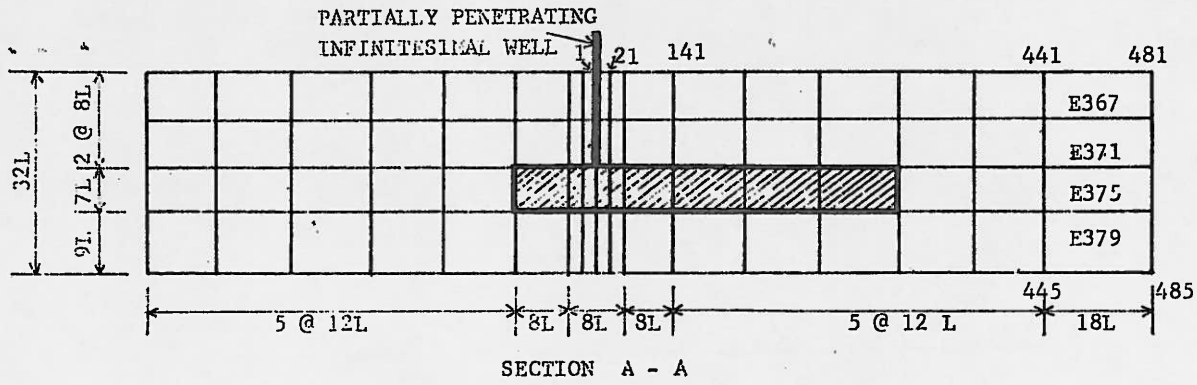


Fig. 10. Simulation of Nonhomogeneous Aquifer by Finite Elements.

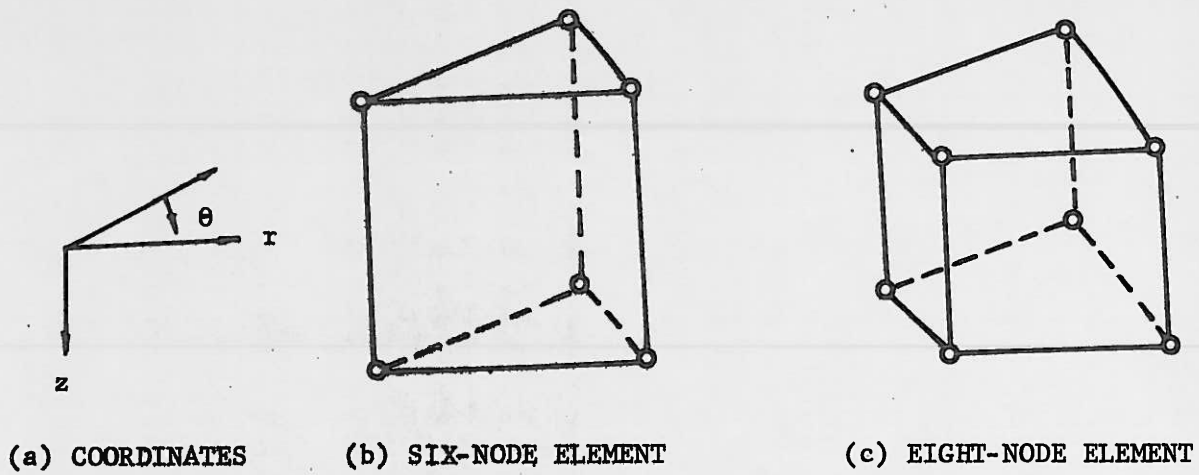


Fig. 11. Three Dimensional Finite Elements.

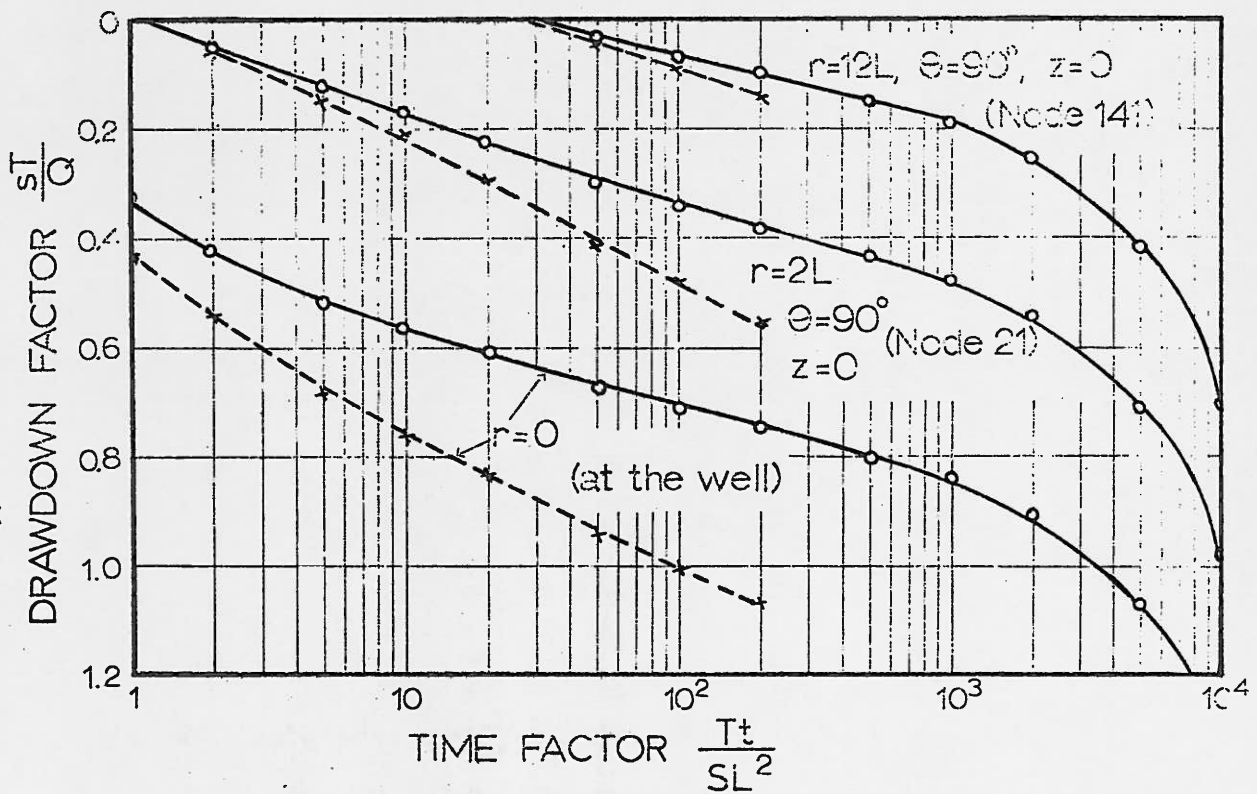


Fig. 12. Drawdown for Homogeneous and Nonhomogeneous Aquifers.

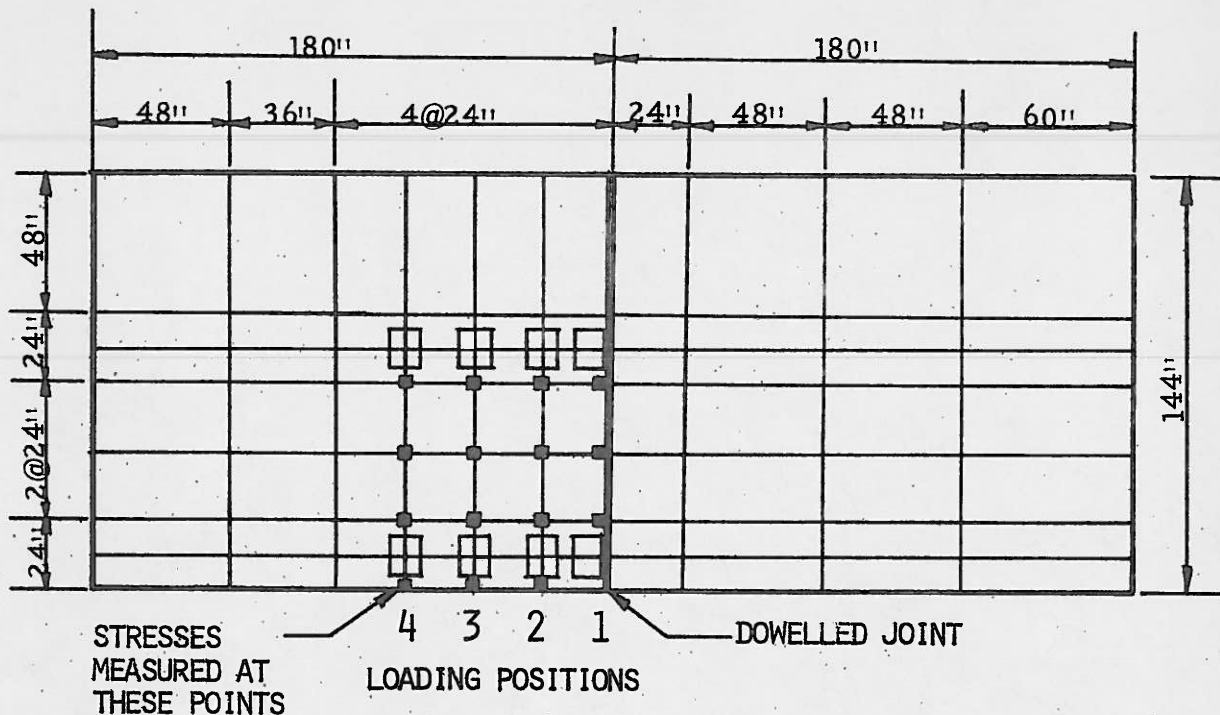


Fig. 13. Loading Positions and Finite Element Grid.

Modulus of  $6.25 \times 10^6$  psi and a Poisson's ratio of 0.28 were used for the concrete. Because of the dynamic nature of the load employed, a large modulus of subgrade reaction of 900 pci was assumed.

Fig. 14 provides a comparison of principal stresses for three cases; viz. 5-in. slab with 12,000-lb axleload, 9.5-in. slab with 22,400-lb axleload, and 12.5-in. slab with 30,000-lb axleload. The finite element solutions are indicated by the solid lines for the major principal stresses and dotted lines for the minor principal stresses. The experimental data are indicated by the small circles. For loading position 1, the stresses are those along the joint. For other loading positions, they are under the respective axle at various distances from the edge.

Fig. 14 shows that the finite element solutions check quite well with the experimental measurements, except for the minor principal stress in the 5- and 9.5-in. slabs. It is believed that this discrepancy is due to the warping of slabs, because the experimental data were taken during the early morning hours when the corners and edges of the slab were warped upward and part of the slab was not in contact with the subgrade, whereas the finite element solution is based on full contact. Even when this warping effect was not considered, the agreement in stress distribution between theoretical and experimental data is certainly surprising and clearly indicates the applicability of the method for predicting the stresses in concrete pavements.

A more general computer program was developed recently at the University of Kentucky for analyzing four slabs interconnected by a longitudinal and a transverse joint. The program can compute the stresses and deflections in concrete pavements due to the combined effect of loading, warping, and nonuniform subgrade support. The solution of this problem could not have been possible without the availability of the finite element method.

#### SUMMARY AND CONCLUSIONS

The finite element method offers a practical technique for solving various problems in geotechnics and transportation engineering. Problems in both structural and nonstructural types can be solved with a minimum of over-simplifying idealizations. The major advantages of the method stem from its simplicity in handling nonhomogeneous, discontinuous, and nonlinear media with irregular shape and cross section, which are typical of natural geologic and soil formations. Because of the large amount of computer time required to solve a realistic problem, care must be exercised in selecting the input data for finite element analyses. It is certainly a waste of time and effort to produce a vast amount of data which are based on unrealistic assumptions. To

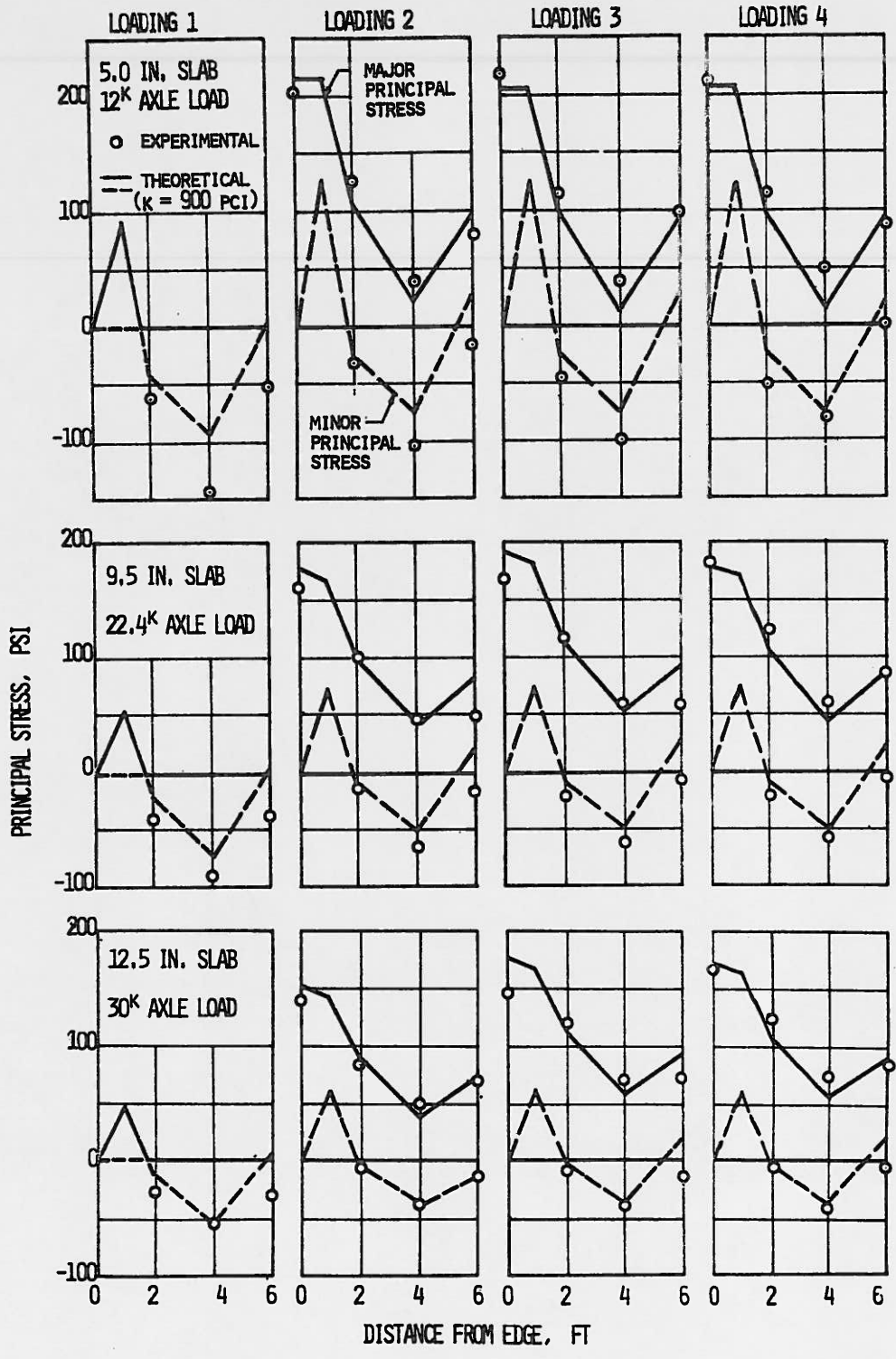


Fig. 14. Comparison of Theoretical and Observed Stresses.

explore fully the power of the finite element method, more research should be directed toward the development of constitutive laws for describing material properties.

The finite element method can be used in geotechnics and transportation engineering as a tool for design and analysis, evaluation of existing structures, post-failure analysis, monitoring and control of construction, location of instrumentation, analysis of laboratory specimens, and field evaluation of material parameters. Although a number of general purpose computer programs based on the finite element method have been developed for problems in structural and continuum mechanics, no such general programs in geotechnics and transportation engineering are currently available. To promote the use of the finite element method, the development of a general computer program such as GEONAP proposed in the Vicksburg conference is urgently needed.

Three examples are given to illustrate the applications of the finite element method. The analysis of nonlinear soil media and the stresses in rigid pavements are problems of the structural type, one in geotechnics and the other in transportation engineering. The determination of drawdown around an artesian well is a problem of the nonstructural type. Because of the complex nature of these problems, they cannot be solved theoretically by the conventional methods, so the finite element method was employed.

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# RELIABILITY ANALYSIS AND COST OPTIMIZATION OF EMBANKMENTS

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## INTRODUCTION

Practically every transportation system in use today has primary or support facilities which require some modification of the natural terrain. The problems presented by terrain modifications involve earth moving, compaction and drainage performed in such a way that the final modification has adequate safety against failure, can be maintained economically, and is constructed at minimum cost. The classical problems faced by soils engineers are intimately related to this process and answers must be developed for typical problems, such as, the optimum slope for a highway or railway cut; the best geometry and method of construction for an embankment; or the expected amount of hard rock or soft soil tunnelling.

In developing designs for earth structures or choosing equipment and methods for tunnelling, experience and precedents are extensively used. In some cases trial construction is undertaken to determine economically suitable geometries or methods. For example, many miles of highway embankments and cuts were constructed by the Ontario Ministry of Transportation. As experience with costs of construction and maintenance grew over the years, the embankment geometries were optimized to reflect the uncertainties and the risks designers were willing to take. Experience is a very important part of soil engineering simply because uncertainty exists in methods of analysis, methods of construction, soil properties, geologic profiles and a host of other factors which influence the performance of earth construction. Consistent means of interpreting uncertainty and incorporating the degree of uncertainty into design decisions has not yet become common practice.

The performance and safety of earth construction is usually evaluated by the resulting movements and the factor of safety. One way of consistently treating the uncertainty associated with estimating these quantities is to use techniques of probability analysis. In the strictest sense, full probability analysis (see Benjamin and Cornell, 1970) may prove to be impractical since data are rarely ever available to define the probability distribution functions of uncertain quantities. It is also

convenient to combine the results of data collection with experience to arrive at best estimates of pertinent quantities. This means that subjective and objective probabilities are combined. In fact, some uncertain factors such as the influence of construction methods may be estimated only subjectively and updated through a collection of performance records. In addition, cost factors and estimates of variability may be incorporated into trade off studies to indicate among other things the expected gain by further data collection, improved methods of analysis, or improved construction methods (Tang, 1972).

## UNCERTAINTY ANALYSIS AND EXPECTED VALUE DECISIONS

A simple first order uncertainty analysis (Cornell, 1971) will be described to demonstrate how uncertainty in the factors which contribute to safety in an embankment influence the uncertainty of the factor of safety. Similar procedures can be applied to settlement, lateral movements or any other quantity that must be predicted before construction.

Uncertain quantities should be represented by at least a best estimate and some measure of the expected dispersion about the best estimate. An uncertain quantity may be treated as a random variable represented by a probability distribution function as shown in Figure 1 for undrained shear strength.

The probability distribution function is  $f_s(s)$  and

$$P(S \leq S_1) = \int_{-\infty}^{S_1} f_s(s) ds \quad (1)$$

is the probability that the undrained strength  $S$  is less than or equal to some specified value of undrained strength  $S_1$ , where

$$f_s(s) \geq 0$$

and

$$\int_{-\infty}^{\infty} f_s(s) ds = 1.$$

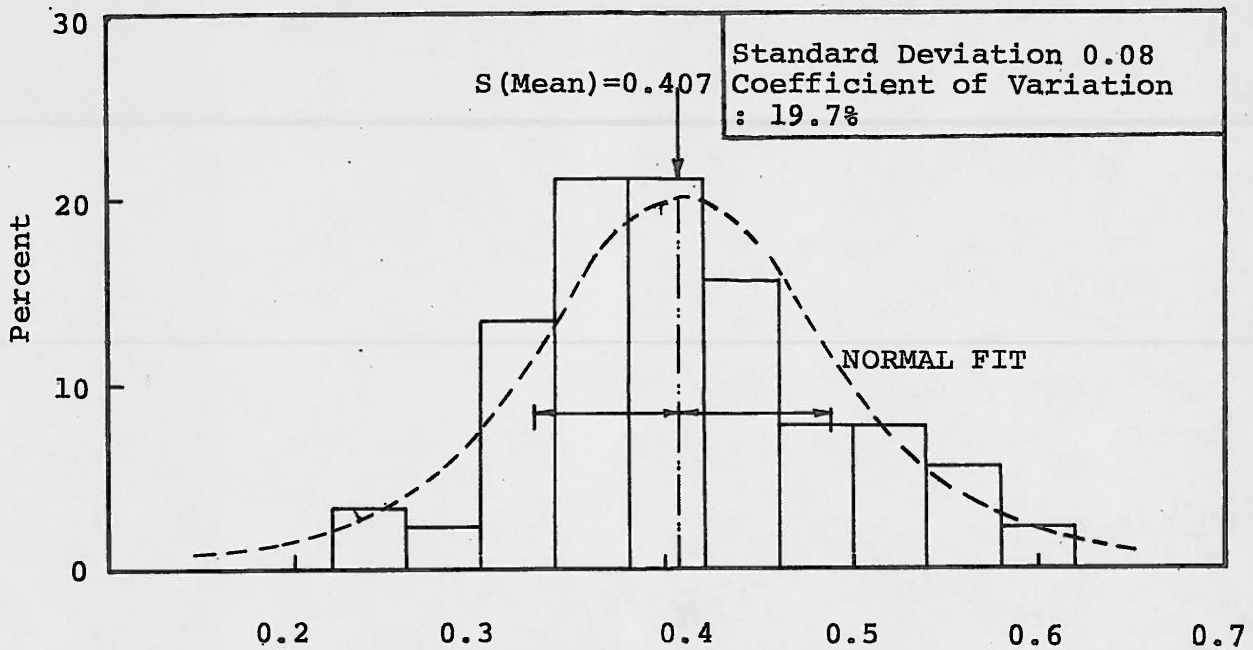


Fig. 1. Histogram of Shear Strength (TSF).

As previously indicated, a full probability analysis may be impractical. Thus, it is advantageous to describe the uncertain quantities by their expected values (mean values) and variances, which can be used for best estimates and measures of dispersion.

The mean value is

$$m_s = E[S] = \int_{-\infty}^{\infty} sf_s(s)ds \quad (2)$$

and the variance is

$$\sigma_s^2 = \text{VAR}[S] = \int_{-\infty}^{\infty} (S - m_s)^2 f_s(s)ds \quad (3)$$

The square root of the variance is the standard deviation and a convenient measure of dispersion for engineering use is the coefficient of variation

$$V_s = \sigma_s/m_s \quad (4)$$

In simplified form, we might evaluate the mean value of shear strength for a soil layer from the average determined from a number of samples,  $n$ . The sample variance would then be evaluated as

$$\text{VAR}[S] = [1/(n - 1)] \sum_{i=1}^n (S_i - m_s)^2 \quad (5)$$

In addition to this sort of uncertainty, testing bias, insufficient sampling, etc. may influence the estimated

mean value; and uncertainty in the mean will also exist. This can also be handled in a similar way and the reader is referred to Benjamin and Cornell (1970) for further details.

The coefficient of variation is a convenient way to express subjective uncertainty (influence of experience) and may be arrived at using some forms of gaming theory or formal assessment of an engineer's judgement regarding particular uncertainties (Tribus, 1969; Taiffa, 1968).

In determining the best estimate of the factor of safety, the best estimates of all factors contributing to resistance and loading should be used. The uncertainty in the factor of safety can be derived from the uncertainties of the contributing quantities. If the factor of safety is given by some model based on bearing capacities or slip surfaces represented by  $g(x_1)$  where  $x_1$  represents the set of uncertain quantities such as shear strength, loads, pore pressure, etc., then the expected value of the factor of safety is

$$E[FS] = E[g(x_1)] \doteq g(m_{x_1}) \quad (6)$$

The result is exact if  $g(x_1)$  is a linear function of the uncertain quantities,  $x_1$ , and a first order approximation if  $g(x_1)$  is not linear. The variance of the factor of safety is given by (Cornell, 1971)

$$\sigma_{FS}^2 = \sum_{i=1}^n [(\delta g / \delta x_i) \Big|_{m_{x_i}}]^2 \sigma_{x_i}^2 + 2 \sum_{j=1}^n \sum_{i=j+1}^n \rho_{ij} \sigma_{x_i} \sigma_{x_j} [(\delta g / \delta x_i) \Big|_{m_{x_i}}] [(\delta g / \delta x_j) \Big|_{m_{x_j}}] \quad (7)$$

$\rho_{ij}$  is the set of correlation coefficients for the uncertain quantities, and they lie between minus one and one. This means that the variance of the factor of safety is influenced upward if, for example, the uncertain quantity of friction angle in one layer is positively correlated with the friction angle in another layer. Positive correlation implies that values of friction angle higher\* than the mean are likely to exist in the same area of both layers. If no correlation exists the dispersion in the factor of safety is directly related to the sum of the variances of the uncertain quantities.

From the best estimate of the factor of safety and the variance of the factor of safety a *reliability index* can be defined as

$$\beta = (E[FS] - 1) / \sigma[FS] \quad (8)$$

The reliability index is a measure of safety as demonstrated in Figure 2. It simply shows the number of standard deviations the expected value of the factor of safety is above a factor of safety of one. In Figure 2a, b and c, the uncertainty in the factor of safety is

\*or lower, i.e. random variables depart from the mean in the same direction.

the same, as indicated by  $\sigma_{FS}$  being equal for all three cases. The probability of failure can be represented by the shaded area and is shown to decrease as the expected value of the factor of safety increases. In Figure 2d and e, the expected value of the factor of safety is the same, but the uncertainty is greater in case e than d, hence, the probability of failure of case e is greater than case d even though they have the same factor of safety.

A decision on the factor of safety applicable to a project can be formulated from an uncertainty analysis using the reliability index. A minimum reliability index seems to be a better specification for safety than factor of safety alone. The influence of uncertainty on the factor of safety for a design based on a specified reliability index is shown in Figure 3.

The uncertainty analysis up to this point does not depend on fitting or choosing probability distribution functions. The reliability index is an improved measure of safety and realistic ranges of this index reflecting professional responsibility and economic considerations should be established. However, this does not provide information on the probability of failure without some probability distribution function. In reality the probability of failure has very little meaning since it is necessary to establish permissible probabilities of failure which would be based on correlations of performance with prediction techniques. Generally acceptable probabilities of failure are quite small and the different distribution functions which may fit data will yield widely different results, i.e. sufficient information to define the tails of the distribution functions is rarely

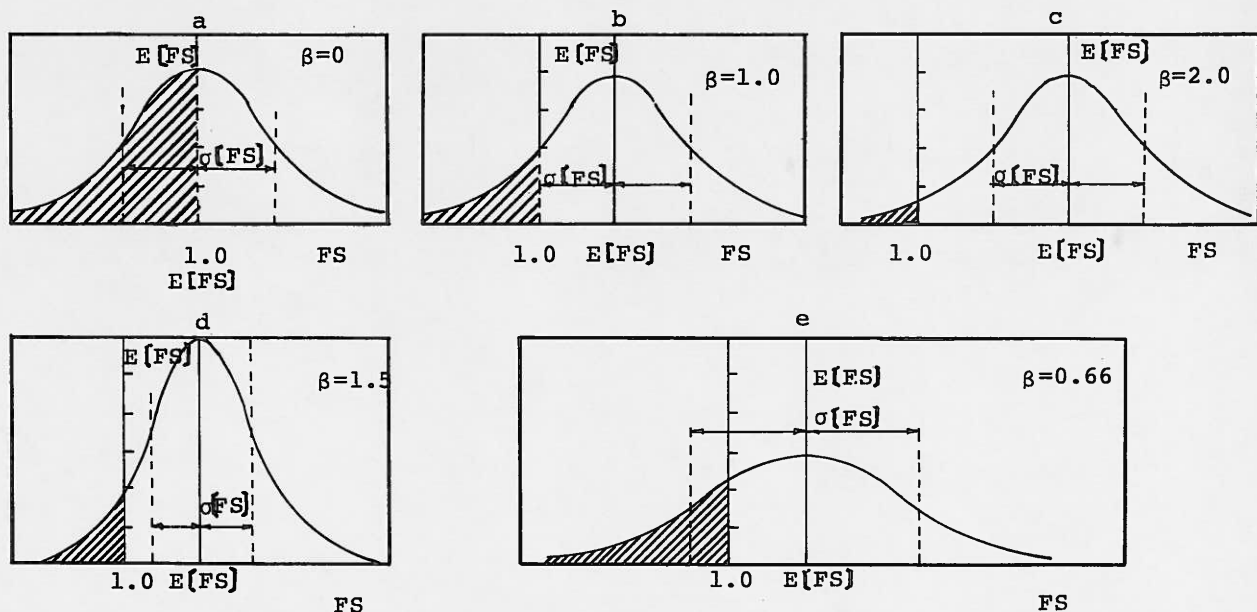


Fig. 2. Graphical Interpretation of Reliability Index.

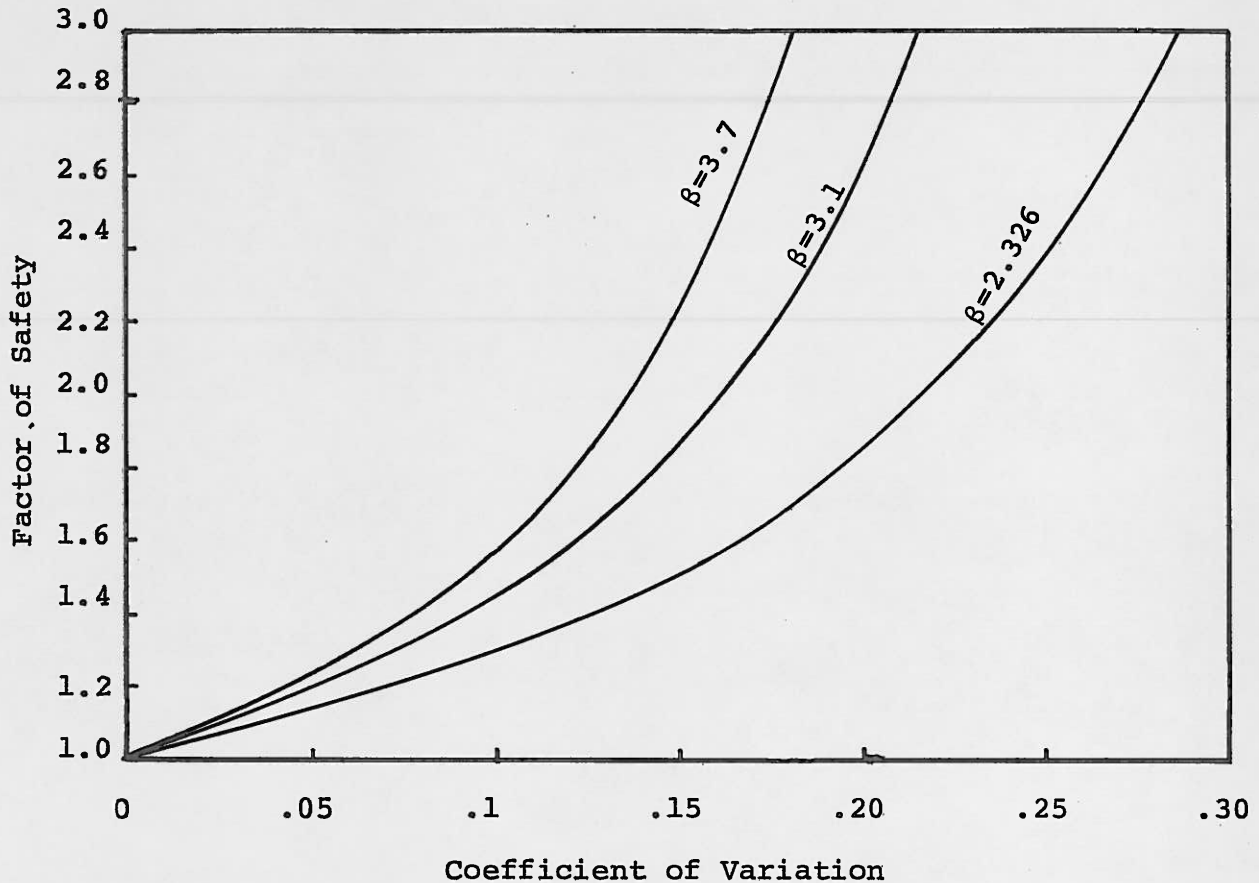


Fig. 3. Factor of Safety Versus Coefficient of Variation for Equal Reliability.

available and the proper function cannot be established with adequate confidence.

Relatively mild assumptions about the probability distribution of the factor of safety or movements, however, will provide a useful means of evaluating the relative benefit of spending money to reduce uncertainty and choosing an economically optimum factor of safety. This is accomplished by minimizing the expected cost (Benjamin and Cornell, 1970; Rosenbleuth, 1969; Resendiz and Herrera, 1969) of the project formulated as

$$E[C] = C_i + C_f P_f \quad (9)$$

where  $E[C]$  = expected cost,  
 $C_i$  = initial cost,  
 $C_f$  = cost of failure, and  
 $P_f$  = probability of failure.

To decrease the probability of failure, i.e. increase the reliability index, the factor of safety must be increased or the uncertainty in the factor of safety (coefficient of variation) must be decreased. Increasing the factor of safety increases the initial construction cost.

Decreasing the uncertainty will also increase the initial cost through the cost of added investigation, cost of greater construction control, or among other things the cost of building and proof testing trial embankments in the case of slope stability.

The expected cost analysis then reflects the interaction of various aspects of the decision process as well as the sensitivity of the cost functions to reductions in uncertainty. For example, the expected cost analysis will indicate how much benefit or loss is derived from taking more borings and running more tests.

The added work increases the initial cost but uncertainty in other factors may override the uncertainty in shear strength and very little change in the reliability index will result. The net effect could be insufficient change in the expected cost, i.e. hypothetical probability of failure, to warrant further investigation.

Establishing the cost of failure is seldom straightforward. Failure is interpreted differently by different clients and owners to reflect their utility in a given project. *Minimum safety criteria must be imposed by the engineer whenever public safety and*

welfare are concerned. Beyond this, there are many advantages to close engineer-owner relationships in evaluating the risks arising from uncertainty and the relative benefit derived from reducing risks or taking risks at specified levels. Since it is the owner's money in the project, it may be more appropriate that he participate fully in the decision process which establishes the level of risk in a project. This approach provides a rational basis for decision making which reflects the cost of averting risk. Some precedence for this exists (Wiggins, 1972) and much more thought is being given to this as new building codes are developed (Esteve and Rosenbleuth, 1971).

Figure 4 is an example of an expected cost analysis on a sand embankment used for fluid retention (Barboteu, 1972). The expected premium cost is defined as the difference in cost for a slope inclination,  $\alpha_0$ , for a factor of safety of one and a slope inclination,  $\alpha$ , less than  $\alpha_0$ . As the slope decreases the factor of safety

increases and the expected premium cost is a savings which reaches an optimum value. The two curves represent a modified form of Equation 9 with different assumptions about the cost of failure. Using the same cost per unit for embankment material and filter material, with a standard deviation of  $3.5^\circ$  for the friction angle and 1.3 feet for the location of the phreatic surface, curve A represents a higher cost of failure than curve B, hence a higher factor of safety is economically justified.

#### ANALYSIS OF A CASE HISTORY

A particularly good case history of a highway embankment failure was reported by Haupt and Olsen (1972). The idealized section is shown in Figure 5 along with parameters for alternate designs. Soil strength testing was limited to laboratory vane shear,

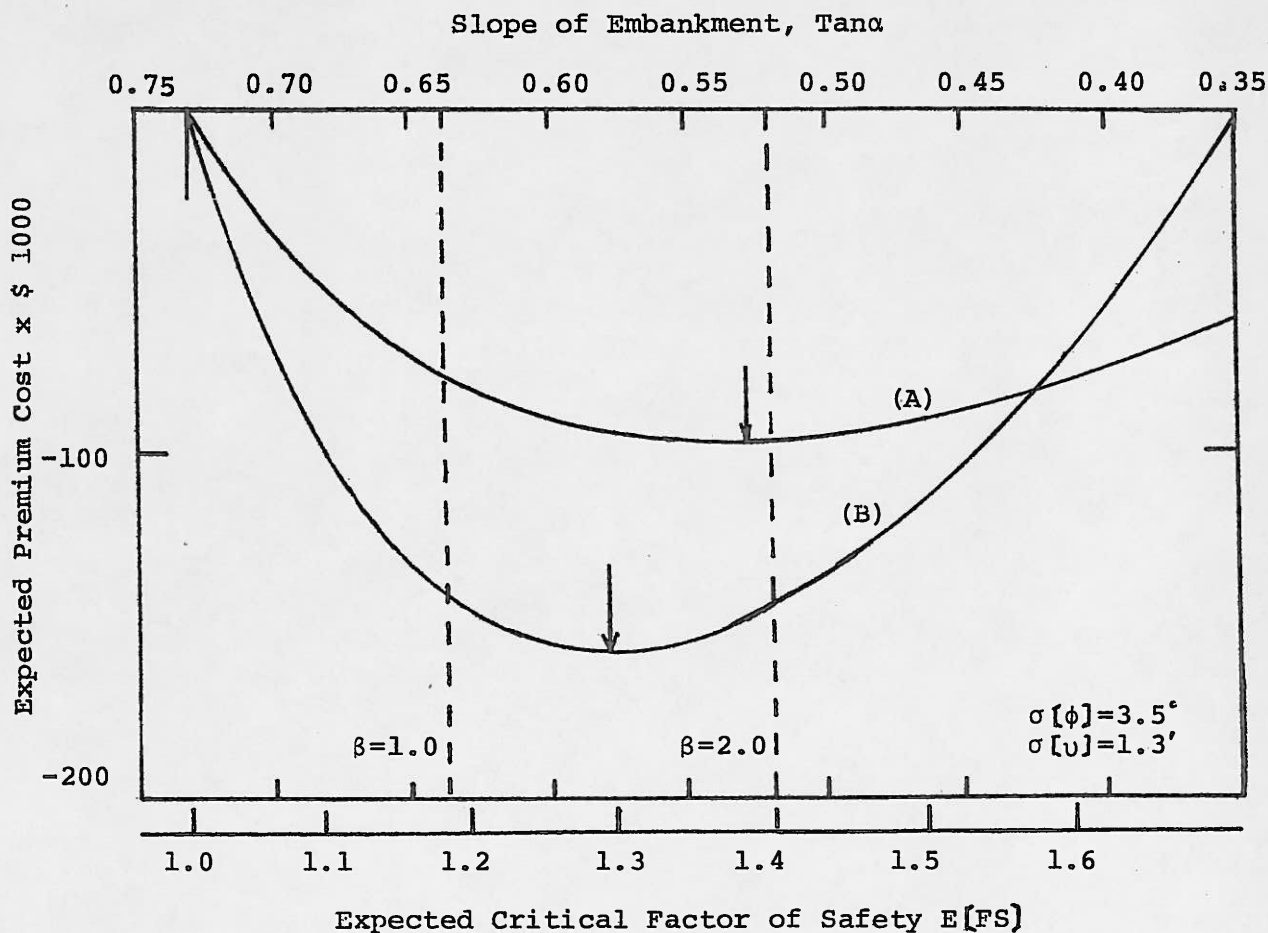


Fig. 4. Typical Shape of the Variation of the Expected Premium Cost Versus Type of Design.

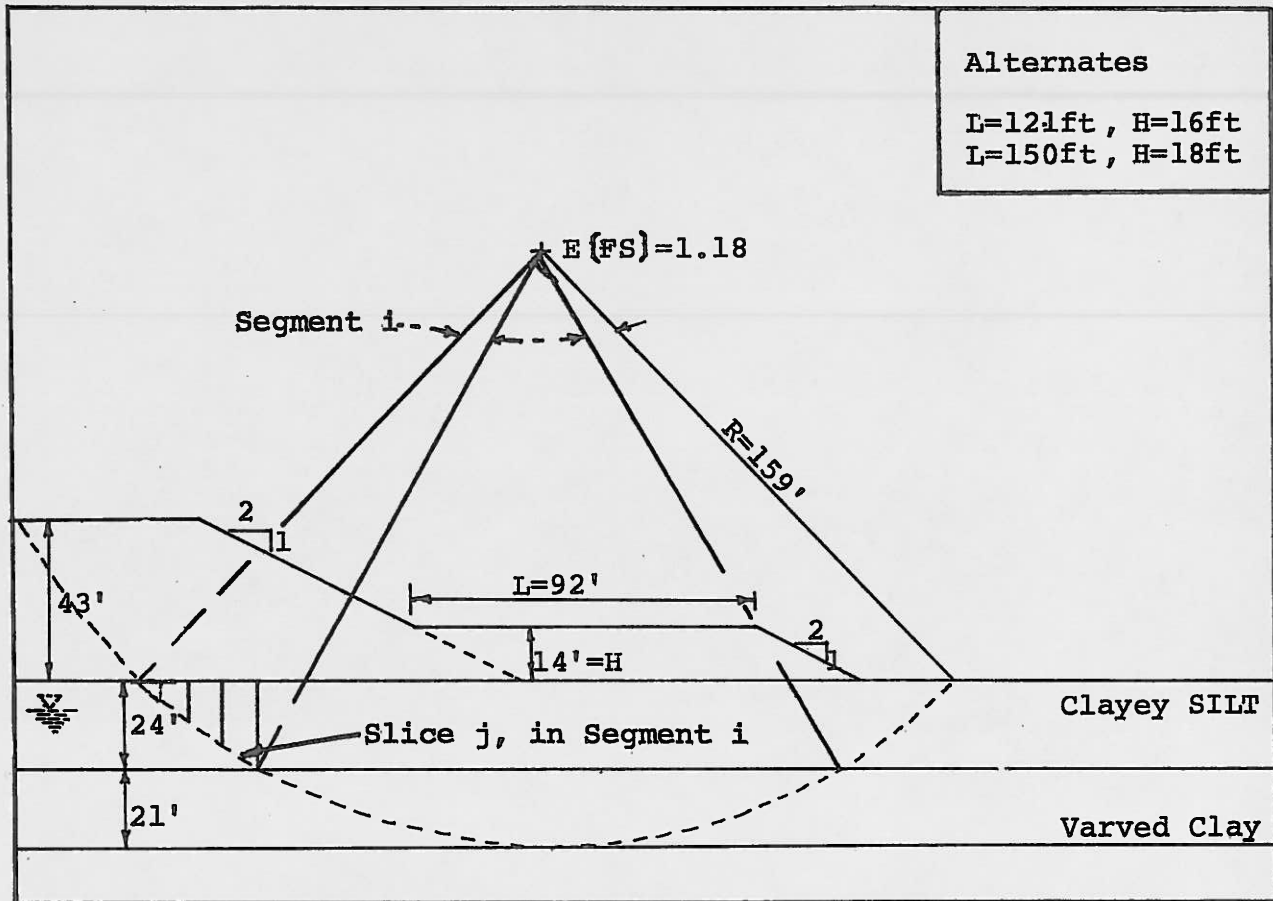


Fig. 5. Fair Haven - Design Section No. 1 and Alternate Designs (Haupt and Olsen, 1972).

unconsolidated undrained triaxial tests and consolidated undrained triaxial tests on two Shelby tube samples. The friction angle of the silt was assumed to be  $30^\circ$  and the laboratory vane shear and unconsolidated undrained triaxial tests gave undrained shear strengths of 750 psf and 790 psf, respectively.

A reliability analysis of slope stability was performed for this case using the modified Fellenius method of slices (Barboteu, 1972). This analysis presents a method of decision making based on limited data.

In the analysis the factor of safety is determined by the ratio of the resisting moment effect to the actuating moment effect. The soil is divided into segments which represent portions of the slip surface that passes through the same soil, i.e. same layer. Each segment is divided into slices based on the size and shape of the segment as shown in Figure 5. The factor of safety can be expressed by

$$(K)FS = \frac{\sum_i [c_i L_i + \tan \phi_i (W_i - A_i)]}{\sum B_i} \quad (10)$$

where  $K$  = influence gravity forces,  
 $c_i$  = cohesion along segment  $i$ ,  
 $\phi_i$  = angle of internal friction along segment  $i$ , and  
 $(W_i - A_i)$  = influence of effective normal stresses along segment  $i$ .

If the soil unit weight is not considered random and no correlation exists between the pore pressure and friction angle then the expected value of the factor of safety and the variance of the factor of safety can be assumed as

$$K E[FS] = E[\sum B_i] = \sum E[B_i] = \sum_i E[c_i] L_i + E[\tan \phi_i] (W_i - E[A_i]) \quad (11)$$

and

$$K^2 \text{VAR}[\text{FS}] = \text{VAR}[\sum_i B_i] = \sum_i \text{VAR}[B_i] + 2 \sum_{i=1}^n \sum_{k=i+1}^n \rho_{ik} \sigma_{B_i} \sigma_{B_k} \quad (12)$$

where  $\rho_{ik}$  represents the coefficients of correlation among the contributions to the resisting moment in the segments.

A computer program was developed (Barboteu, 1972) to evaluate the expected value of the factor of safety, the reliability index and the expected cost, given the best estimates and measures of uncertainty of the design parameters. For this case the best estimate of the undrained shear strength of the varved clay was 770 psf and the friction angle of the silt was 30°. The elevation of the ground water table was taken as 8.5 feet below existing grade. No information about the uncertainty involved with these design parameters was available, hence subjective estimates by the engineer are necessary.

Unconsolidated - undrained triaxial tests and vane shear tests involve a great deal of uncertainty (Ladd, 1971) and variability of strength parameters over the site add to this uncertainty. Coefficients of variation for both friction angle data (Singh and Lee, 1971) and undrained shear strength (Lumb, 1966, Ladd, et al., 1971) have been evaluated and may be used to get some estimate of the expected dispersion. Since the uncertainty must be evaluated subjectively, at this point an engineer's experience can be formally inserted into the reliability analysis. Firsthand knowledge of the site, results of tests in nearby deposits and any other forms of information will aid the engineer in arriving at best estimates of the parameters and the degree of confidence he has in these best estimates reflected in choosing a coefficient of variation.

The results of stability analyses for different alternatives of berm design based on the best estimates previously cited are shown in Figure 6.

Introducing uncertainty into the analyses, the minimum reliability index was computed for different berm designs approximated by the cross-sectional area of the berm. The results are shown in Figure 7 for various coefficients of variation of the undrained shear strength. The standard deviation of the friction angle was chosen as 4.5° and the standard deviation of the ground water elevation was taken as 4 feet. These standard deviations were estimated from typical variations found in the friction angle and expected seasonal fluctuations of the ground water table. Figure 8 shows the influence of the uncertainty in the undrained shear strength on the factor of safety required to attain a particular reliability index. If it is decided that highway embankments should be constructed with

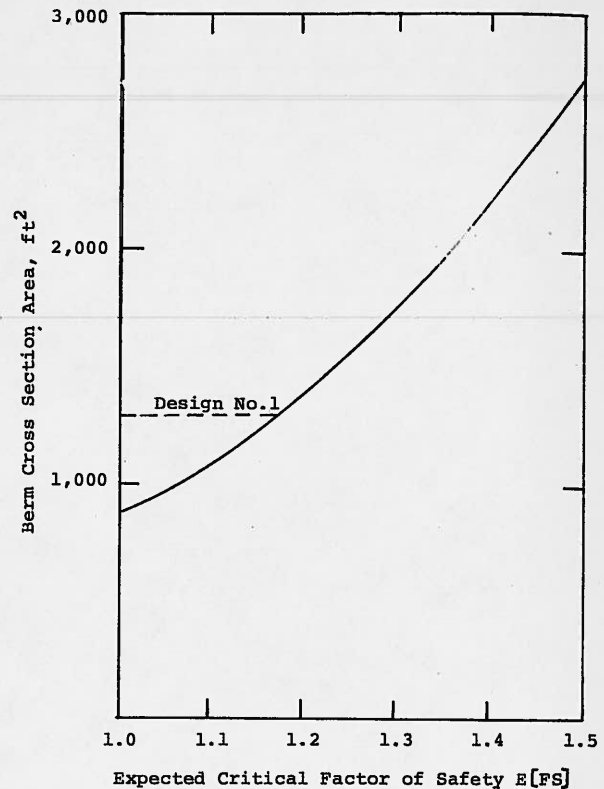


Fig. 6. Expected Critical Factor of Safety Versus Berm Design.

a reliability index of two, for example, then the factor of safety required, and in this case the extent of the berm, is determined by how well the undrained shear strength can be estimated, given the uncertainty already established for the friction angle and ground water level.

Figure 8 indicates that the factor of safety would have to be increased from 1.26 if the coefficient of variation of the undrained shear strength is ten percent to 1.45 if the coefficient of variation is 25 percent.

Using an expected cost analysis (as outlined in the section in UNCERTAINTY ANALYSIS AND EXPECTED VALUE DECISIONS) provides a rational basis for choosing the factor of safety. This required some assumption about the probability distribution of the factor of safety. Over the range of the expected value of the factor of safety plus or minus two standard deviations, normal distribution law may be an adequate representation for purposes of expected cost analysis. Table I shows the relationship of the reliability index and the hypothetical probability of failure for the normal distribution law.

An initial cost is associated with each factor of safety and is based on the cost of placing embankment and berm material. The cost of failure during construction may represent loss of time and

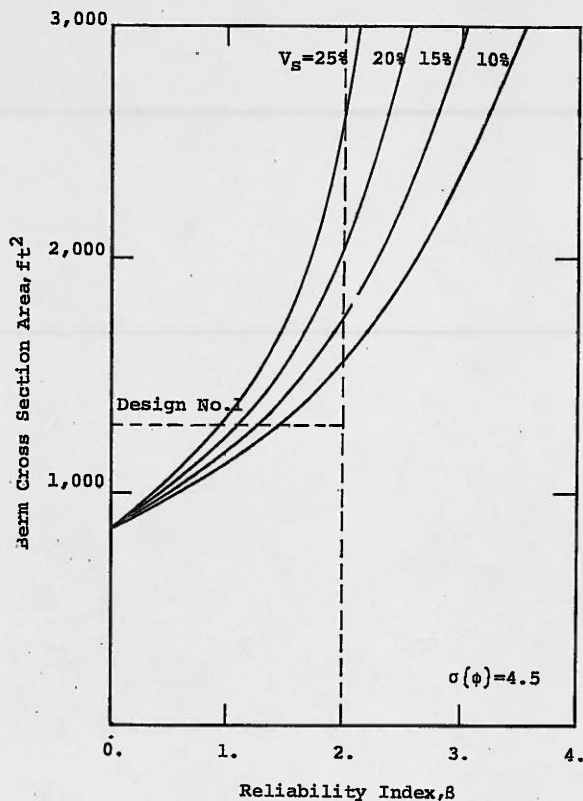


Fig. 7. Reliability Index for Critical Circle Versus Berm Design and Coefficient of Variation of Clay Strength.

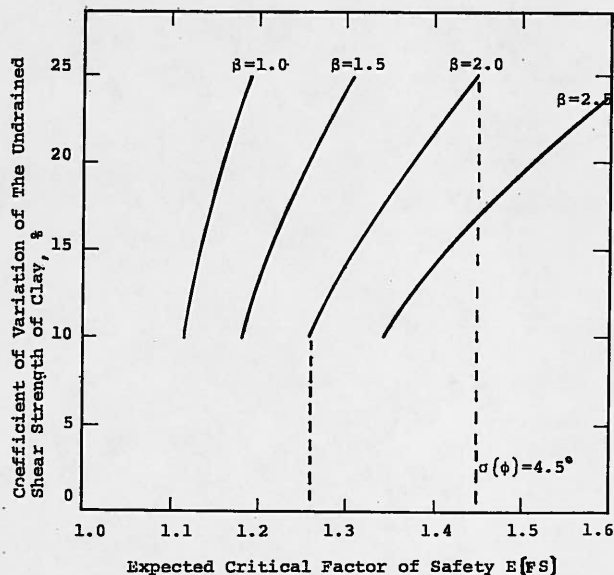


Fig. 8. Curves of Equal Reliability Index.

TABLE I

NORMAL DISTRIBUTION LAW

RELIABILITY INDEX $\beta$	PROBABILITY OF FAILURE $P_f$ , %
0	50.0
1.0	15.9
2.0	2.3
2.32	1.0
3.1	0.1

reconstruction costs. If failure occurs after the roadway is open, the cost of failure may be increased due to rerouting traffic, negotiation of a new contract for repair, and loss of highway utility for a period of time.

A parameter study of the economically optimum factor of safety for various unit costs of berm fill material and costs of failure was made. The results of the expected cost analyses are shown in Figure 9 for an undrained shear strength coefficient of variation of 25 percent.

Design I was actually constructed and failed. The cost of reconstruction was \$97,000. No information was available on the unit costs of original construction.

CONCLUSIONS

The preceding simple reliability analysis demonstrates a method for making decisions about the factor of safety which reflects the uncertainty faced by soils engineers. Since construction and maintenance costs may be closely related to the factor of safety, the cost implications should be considered when a design factor of safety is chosen. One way of reflecting the cost, which must also include the consequences of failure, i.e. maintenance, reconstruction, etc., is to evaluate a relative probability of failure. The probability of failure multiplied by the cost of failure when added to initial costs gives an expected cost which reflects the trade off between initial cost and suitable risk.

In many projects it then becomes possible to discuss alternatives with the owner using a format he can understand and may very well use in his own business for making decisions.

This increases the understanding between the engineer and owner, and gives the owner an appreciation for the uncertainty involved in the design and performance predictions. Since the owner's money is



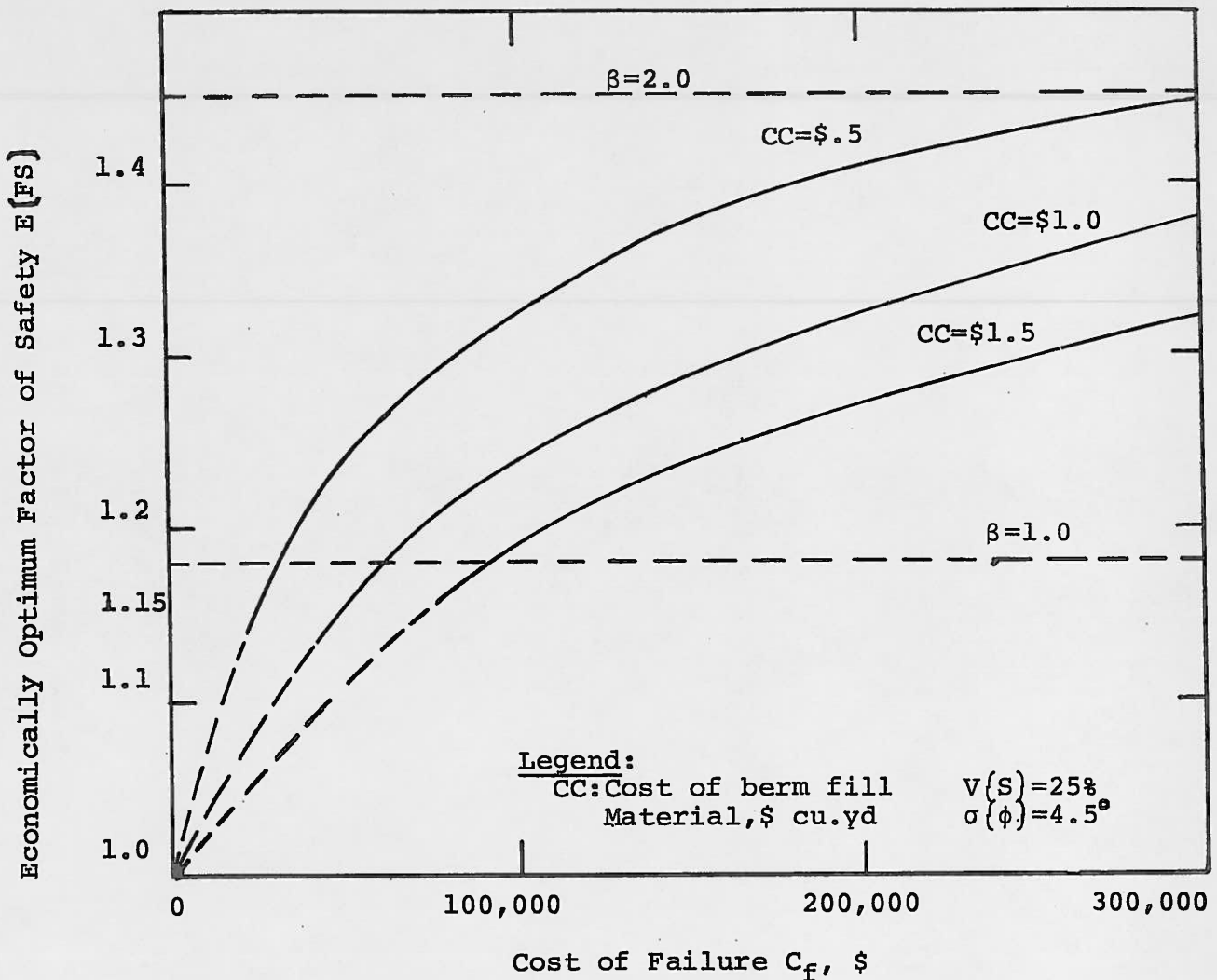


Fig. 9. Economically Optimum Factor of Safety vs Cost of Failure.

involved he then has a format for translating the risk he is willing to take into dollars. The engineer, of course, still has the responsibility for controlling the maximum acceptable risk to guard public safety.

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**SOIL ENGINEERING FROM THE  
HIGHWAY ADMINISTRATOR'S VIEWPOINT**

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## STABILITY ANALYSIS -- ITS POTENTIAL AND ITS LIMITATIONS

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The early developments of stability analysis some sixty years ago marked the beginning of modern soil mechanics. Since that time, stability analysis has been widely adopted in the design and construction of slopes and earth structures. Since the methods of analysis are derived from the principles of statics, there is very little doubt about the validity of the basic principles. Our primary concern is whether the material properties and boundary conditions used in the analysis adequately represent the problem at hand. In other words, we must insure that: (a) the measured shear strength is representative of the in-situ shear strength, (b) the subsoil profile is an accurate reflection of the stiff and weak layers that exist in the ground, and (c) other important parameters such as porewater pressure are accurately known.

The most successful stability analyses have been made on slopes in soft to medium, fairly uniform clays. Here the subsoil conditions are known with a good degree of reliability and we can concentrate on the accuracy of our measurement of the shear strength. The circular arc method in its various forms may be used, and the success can best be evaluated through studies of actual slope failures. If the measured shear strength is correct, the computed safety factor should be 1.0. An extensive list of case histories of failures has been compiled by Bishop and Bjerrum (1960) which shows that the computed safety factors of slope failures are generally close to 1. Additional cases of failures have occurred since that time. Considering first, failures immediately after construction, a graphical representation of the computed safety factors is shown in Fig. 1. The term  $v$  denotes the ratio of the observed to the computed safety factors. The figure shows that the vane shear test tends to underestimate the strength. While the unconfined compression test appears to give more satisfactory results, this conclusion should be tempered by the observation that most unconfined compression test samples are disturbed to some extent by sampling. Thus, the agreement may merely reflect a compensation of errors.

To evaluate the reliability of a particular test we should examine the stress conditions in the test relative to those in the field. The stress conditions in a slope failure are shown in Fig. 2. Along the shear zone bc,

the principal axes depart appreciably from the vertical direction and the failure surface is nearly horizontal, a condition which does not exist in the unconfined compression test or the triaxial test (Fig. 3a). On the other hand, the vane shear (Fig. 3b) measures primarily the strength on a vertical cylindrical surface. Another discrepancy lies in the magnitude of the intermediate principal stress. The field situation may be approximated by the condition of plane strain in which the strain perpendicular to the cross-section in Fig. 2 is zero. The plane strain test (Fig. 3c) may be used to reproduce the stress condition along cd of the slip surface, and the Geonor simple shear test seems to provide the best approximation to the stress conditions along bc. The importance of these factors depend to a large extent on the material involved. Limited empirical evidence suggests that the effects of stress conditions are most serious with soft clays of high plasticity.

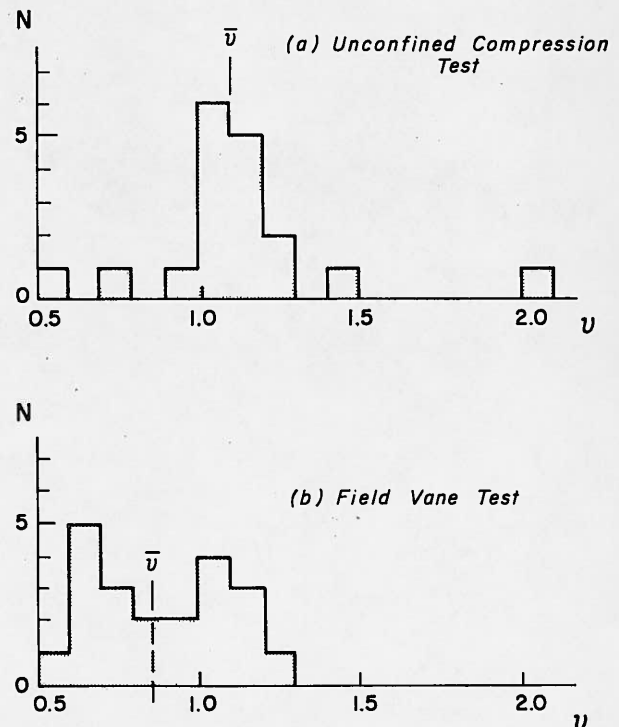


Figure 1.

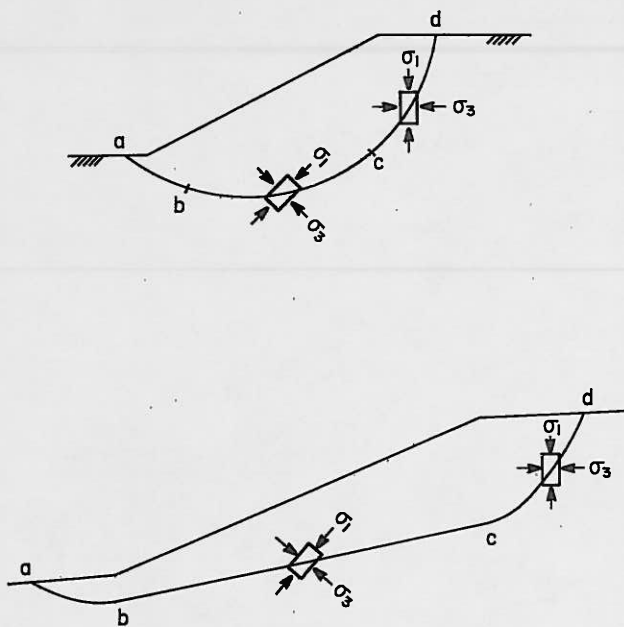


Figure 2.

Since most landslides involve complex subsoil conditions, the uncertainty associated with our knowledge of the subsoil becomes the major factor. This is the case even with soil deposits that may appear to be quite uniform. An example is the embankment near Cleveland (Fig. 4). Initial borings and tests indicated that the subsoil consisted of medium to stiff silty clay and the computed safety factor was around 1.4 based on the results of unconfined compression tests. Yet, when the embankment was built to the final height, a slide occurred. Additional borings revealed a zone of soft clay at around elev. 613 which was missed in the earlier borings and was not present outside the area of the slide. None of the earlier borings was located in the slide area.

The additional borings and several test trenches provided many samples and a thorough investigation of the shear strength was carried out. In addition, the embankment was instrumented with piezometers and slope indicator tubes so that the porewater pressure at the time of the slide and the location of the slip surface were reasonably well known. A stability analysis using the Morganstern-Price Method was made.

The shear strength of the soft layer had a predominant influence on the stability, and the results of the strength tests are summarized in Table 1 together with the computed safety factors. The shear strength measured by the unconfined compression test ranges

between the extremes of 270 and 1000 psf (Table 1). Many of the samples with low strength contain slickensides. The results of the stability analysis indicate that for a safety factor of 1 the shear strength should be around 500 psf. While this value seems reasonable in view of the test results shown in Table 1, we can also visualize the difficulty in choosing the design strength even if all the data in Table 1 were available before construction.

The likelihood that the unconfined compression strength may be influenced by sampling disturbance led to the investigation of the normalized undrained shear strength (Ladd, 1969). The shear strength was measured by the Norwegian simple shear test (Bjerrum and Landva, 1966) and by the triaxial test. Tests were performed on block samples cut out from test trenches. The computed safety factor is between 1.1 and 1.5 using the shear strength measured by the simple shear test and between 1.8 and 3.0 using the shear strength measured by the triaxial test. Thus the simple shear test appears to give reasonably satisfactory answers. This large difference illustrates the important influence of the stress conditions on the strength. However, since only a few block samples were taken, it cannot be assured that these samples are representative of the material in

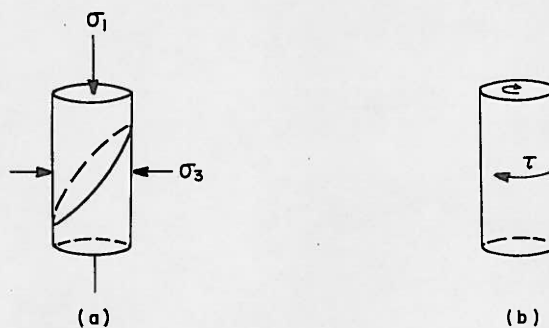
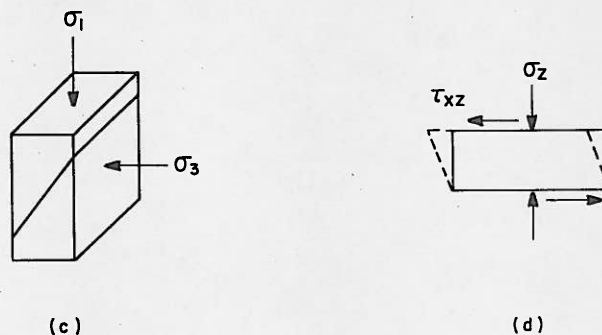


Figure 3.



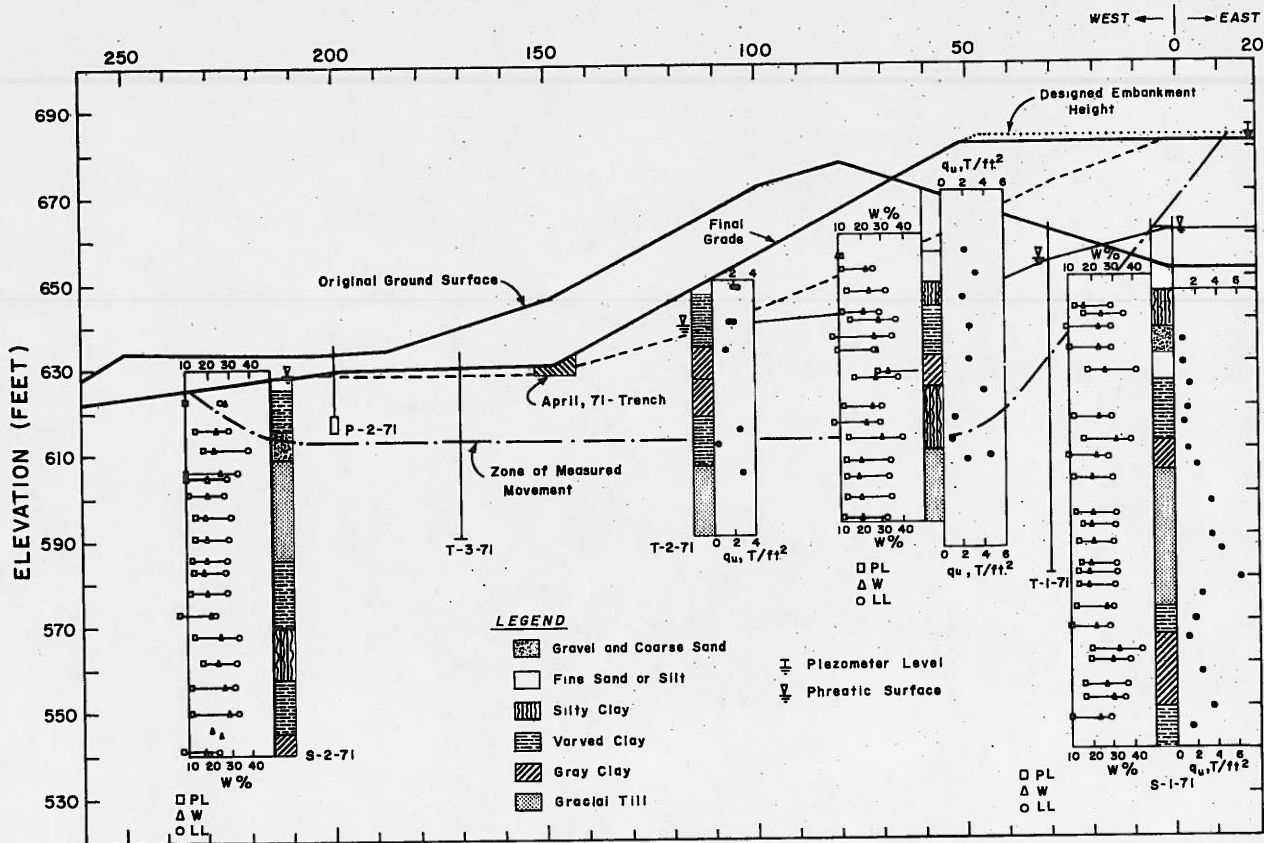


Figure 4.

the failure zone. This example illustrates the difficulty of establishing the reliability of test methods because of the variability in natural soil deposits.

The evaluation of the in-situ shear strength becomes even more difficult in the case of stiff-fissured clays. An example is the colluvial soils of southeastern Ohio derived from weathering of the Conemaugh clay-shale. The soils usually consist of stiff, stoney clay, variably cemented and containing fissures and slickensides. Studies of unstable slopes (Hooper, 1969) that had undergone large movements show that slip occurs over a slickenside surface. The in-situ shear

strength along such a surface is close to the residual shear strength measured in direct shear tests. This is in agreement with the conclusions of Skempton and Hutchinson (1969) on "slides on pre-existing slip surfaces". However, the use of the residual shear strength in analyzing "first time slides" (Skempton and Hutchinson, 1969) could be clearly uneconomical in many cases. The difficulty then lies in the prediction of the in-situ strength of a proposed excavation before a slickenside surface has developed.

The following example of a slope in southeastern Ohio illustrates this problem. The slide occurred shortly

TABLE 1. SHEAR STRENGTH AND COMPUTED SAFETY FACTORS, CLEVELAND SLIDE

METHOD OF MEASUREMENT	SHEAR STRENGTH	SAFETY FACTOR
Unconfined compression test	290 to 1300 psf	0.9 to 1.4
Simple shear test	410 to 1520 psf	1.1 to 1.5
Triaxial test	1270 to 3400 psf	1.8 to 3.2

after an excavation was made (Fig. 5) with movements near the toe and cracks at the top of the cut slope. This was followed by disintegration of the slope into large blocks. Movements eventually progressed uphill for a distance of over 100 ft from the cut slope. The shear strength of the intact clay was about 2400 psf. The strength of the clay that occupied the fissures ranged between 400 and 1000 psf. Exposures in test trenches revealed slickensides in directions that generally paralleled the original ground surface. Therefore, it is likely that this is the site of an old landslide, a feature which has been frequently observed in colluvium (D'Appolonia et al., 1967). The natural slope was inclined at about 12° with the horizontal and the residual shear strength measured in direct shear tests was between 12° and 14°. Nevertheless, the natural slope, prior to the excavation, had been stable for many years.

To evaluate the in-situ strength of the "first time slide", the section shown in Fig. 6 was chosen as representative of the initial slide. Stability analyses were made with (a) residual shear strength along bc and (b) residual shear strength along bcd. For these two conditions the shear strength of the fissured material along (a) ab and (b) cd were found to be 700 and 1050 psf respectively. These values were used to calculate the "residual factor" (Skempton, 1964) defined as

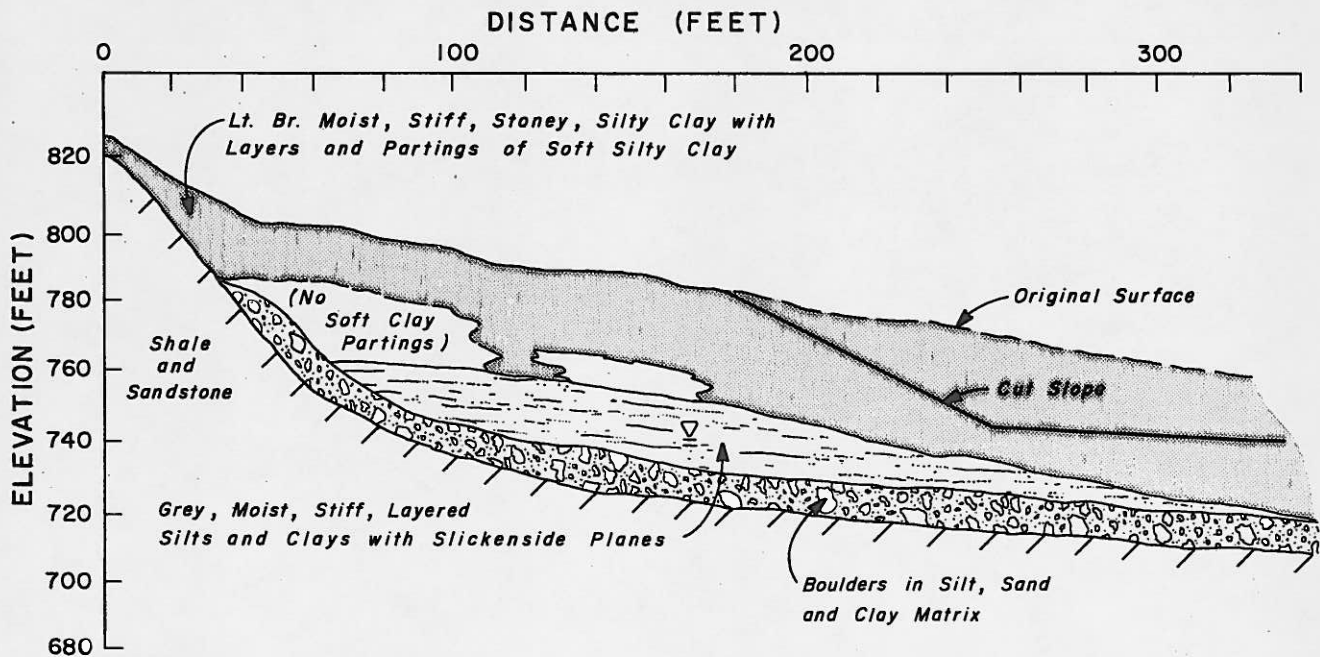
$$R = (s_f - \bar{s}) / (s_f - s_r) \quad 1$$

TABLE 2. COMPUTED RESIDUAL FACTORS IN A STIFF-FISSURED CLAY

CONDITION	SHEAR STRENGTH AT FAILURE ALONG	R
$s_r$ along bc	700 psf	0.85 - 1.00
$s_r$ along bcd	1050 psf	0.68 - 0.96

in which  $\bar{s}$  is the mean shear strength at failure, and  $s_f$  and  $s_r$  are the peak and residual shear strengths. The results are shown in Table 2 and are in the general range of values found by Skempton and Hutchinson (1969) for the "first time slides". From a practical viewpoint, the main difficulties lie in the detection of slickensides and the possibility of progressive failure (Bjerrum, 1967). There is no assurance that the slope would remain stable for long times if the shear stress were kept below  $\bar{s}$ .

A very different situation is encountered in the shallow slides that occur on hillsides in southeastern Alaska. The slides occur most frequently after logging and during the Autumn rain season (Bishop and Stevens, 1964). The slopes on which slides have been reported are inclined at angles between 36° and 41° with the



Subsurface Profile at Brilliant, Ohio, Landslide

Figure 5.

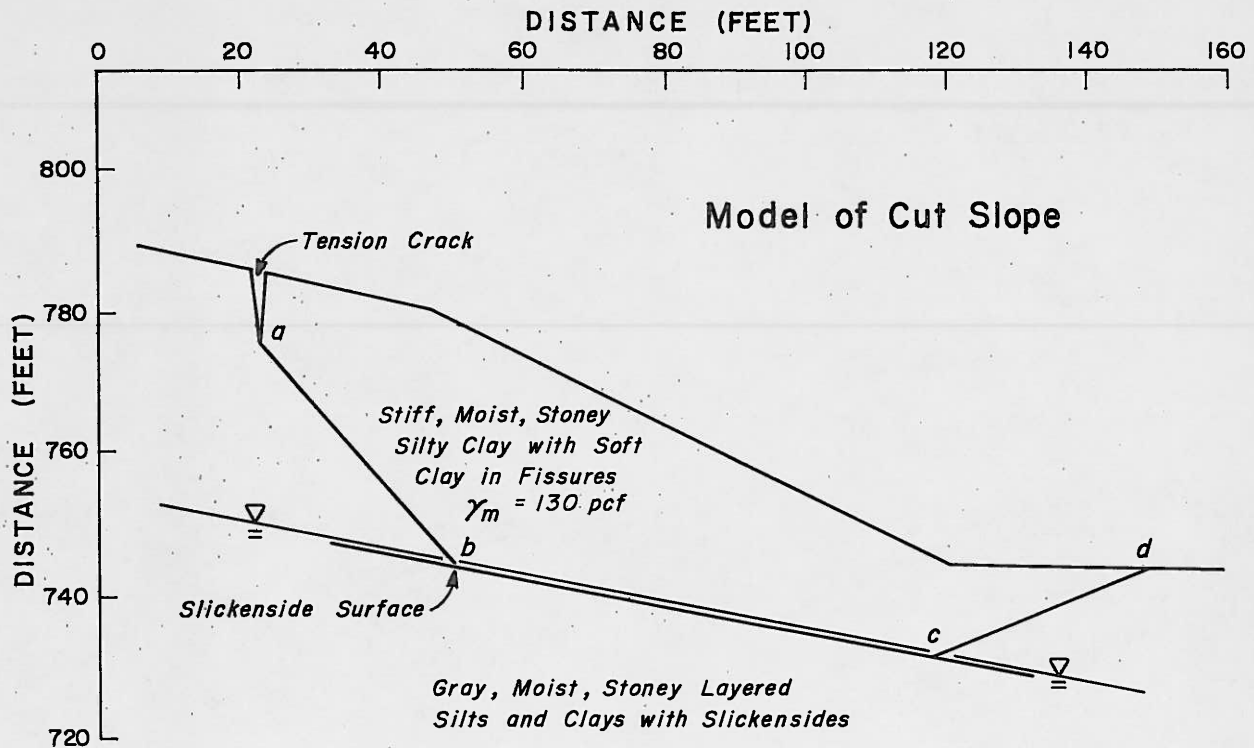


Figure 6.

horizontal. The soil consists primarily of glacial till and weathered till which contain a large percentage of gravel.

Since the failures took place during periods of heavy rainfall and therefore high pore pressure, an effective stress analysis should be made. The shear strength was measured by drained direct shear tests and ranged between the limits of  $\bar{c} = 0$  and  $\bar{\phi} = 38^\circ$  to  $\bar{c} = 150$  psf and  $\bar{\phi} = 36^\circ$ . In this case the in-situ pore water pressure must be determined. The measured pore water pressure is shown in Fig. 7. One can see that it is strongly influenced by the rainfall pattern. The highest pore pressure appears to occur after several continuous days of heavy rainfall. On the other hand, a single day of very heavy rainfall, say over 2 in., if not preceded by several days of heavy rainfall, does not appear to be as important a factor. Stability analyses assuming an infinite slope indicate that the slopes should have failed given the observed maximum pore pressure. However, it is not possible to predict the stability of a slope until a more reliable means for prediction of the pore pressure is established.

The preceding examples in no way diminish the importance of stability analysis. Indeed, it is only after the establishment of the basic laws on mechanics of slope failure and shear strength of soils that we are able to consider the influence of the various factors on stability. The examples serve to illustrate various

complications in natural laws that we cannot presently analyze with much confidence. In Peck's (1967) words, "Nature was able to outwit us and we fear she can and will do so in the future". It therefore behooves us to approach all tampering with natural processes with due caution and respect. It follows also that whenever we venture forth on a large scale, sound engineering requires that careful observations be made on the actual performance so that serious departures from our predicted performance may be detected before the consequences become catastrophic.

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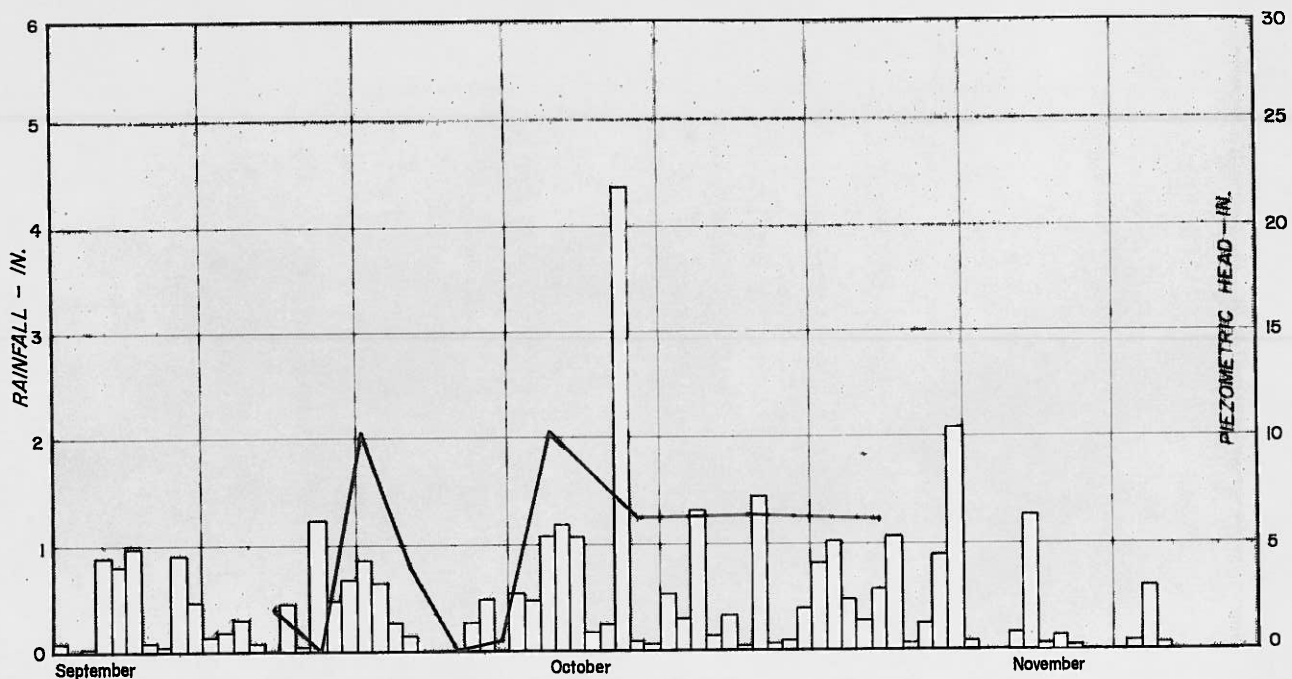


Figure 7.

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